Misallocation due to Incomplete Markets

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Abstract

We quantify misallocation caused by limited risk-sharing and imperfect consumption-smoothing within and across borders. We measure misallocation as the share of resources left over if financial markets are completed and consumers are compensated to maintain their status quo welfare. This measure is a multi-agent extension of Lucas's (1987) consumption-equivalent variation and does not rely on a social welfare function or on interpersonal utility comparisons. We provide exact formulas and deadweight-loss triangle approximations that can be applied without specifying income processes or modeling households' portfolio choices. We find that incomplete risk sharing across US households destroys the equivalent of about 20 percent of aggregate US consumption. Incomplete risk sharing across countries, abstracting from within-country heterogeneity, destroys the equivalent of roughly 5 percent of world consumption. The latter is driven by fast-growing countries such as China and India, while unexploited consumption-smoothing opportunities among countries at similar levels of development are much more limited. Thus, most of the potential gains from more complete markets come from insuring household-level risks domestically and from intertemporal trade between fast-growing and slower-growing countries, rather than from additional risk sharing among similarly advanced economies.

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1 Introduction

The inability of households to perfectly share risks across states of nature or smooth consumption over time is a form of resource misallocation. We quantify this misallocation by asking the following question: if financial markets were completed and every household were compensated so that no one is worse off than under the status quo, how much of the economy's resources would be left over? We characterize this measure of misallocation in a range of workhorse models with incomplete financial markets — both in closed- and open-economy settings. We provide exact formulas for these welfare losses and show how to approximate them using sufficient statistics derived from observed consumption allocations.

The outline of the paper is as follows. In Section 2, we provide a general definition of misallocation following Debreu (1951), and more recently, Baqaee and Burstein (2025b). We measure misallocation by the maximum proportional contraction of the feasible consumption possibility set — or, equivalently, of total factor productivity — that keeps every agent at least as well off as under the status quo allocation. The resulting contraction factor is a measure of the waste due to market incompleteness. This measure is a multi-agent extension of the consumption-equivalent variation defined by Lucas (1987). One of its key properties is that it quantifies the distance from the efficient frontier without making interpersonal utility comparisons (i.e., it is invariant to monotone transformations of utility). The measure also easily accommodates heterogeneous preferences and, unlike social welfare functions, does not conflate inequality with inefficiency. Building on results from Baqaee and Burstein (2025b), we show that this measure of misallocation can be computed by solving a simple maximization problem.

In Section 3, we set up a closed-economy model with idiosyncratic and aggregate risk, but without labor-leisure choice or physical capital accumulation. We show that, in this baseline case, misallocation is equal to the certainty equivalent of the aggregate consumption process divided by the sum of the certainty equivalents of each household's consumption process. In this special case, our measure is similar to the efficiency measure proposed by Benabou (2002), though for us, this formula is a result rather than a definition. Once other ingredients are added, such as labor-leisure choice, our measure differs from Benabou-style measures, and the latter have a counterintuitive preference for more inequality.

We provide a second-order approximation of misallocation in terms of deadweight-loss (Harberger) triangles. This sufficient-statistics formula can be computed using only consumption panel data and values for the elasticity of intertemporal substitution and risk-free interest rate. In particular, we do not need to know the nature of households' portfolio problems, income processes, or ownership of assets.

We apply these formulas to an off-the-shelf calibration of a Bewley (1972) economy. We find that imperfect insurance against idiosyncratic risk is equivalent to a loss of around 20 percent of aggregate consumption. That is, if markets were complete, we could make every agent indifferent to the status quo and have 20 percent of the aggregate consumption good left over in every date and state. This loss is roughly three orders of magnitude larger than Lucas's (1987) estimate of the cost of aggregate volatility in a representative-agent economy.[1] Furthermore, our second-order approximation is extremely accurate in this quantitative example — even though the equilibrium is far from the point of approximation. We also show that the efficiency gains from completing financial markets are roughly half as large as the distance to first-best according to the popular behind-the-veil-of-ignorance social welfare function. This is because the veil-of-ignorance social welfare function penalizes inequality as well as inefficiency.

In Section 4, we extend the simple model to allow for labor-leisure choice and for aggregate capital accumulation. Since our measure of misallocation is defined in very general terms, we can apply the same definition with these additional ingredients. In this case, the consumption possibility set also includes leisure and takes into account the fact that the aggregate capital stock is endogenous. With these features, our measure of inefficiency also captures inefficiencies caused by excessive capital accumulation, due to the precautionary motive, and distortions in labor-leisure decisions. We show that with these ingredients, our measure differs from the measure of aggregate efficiency proposed by Benabou (2002). In particular, we show that the latter has some counterintuitive properties. For example, among points on the Pareto-efficient frontier, the Benabou (2002) measure may strictly prefer points with more inequality once there is labor-leisure choice. By continuity, this also means that Benabou-style measures can prefer Pareto-inefficient allocations to Pareto-efficient ones once leisure is endogenous.

In Section 5, we introduce international trade by generalizing the baseline model to allow for heterogeneity in consumption baskets across households. In particular, we study a setup with multiple countries, multiple industries, and international input-output linkages. We extend our second-order approximation to allow for these features. This ap-

[1]Following Lucas's estimates for aggregate consumption, a series of papers have estimated the gains from eliminating volatility in settings with heterogeneous agents. See, for example, Imrohoroğlu (1989), Atkeson and Phelan (1994), Storesletten et al. (2001), and Constantinides (2025). To aggregate welfare gains across agents, these papers rely either on a social welfare function (e.g., "average utility" or veil-of-ignorance) or have ex-ante symmetry (so that welfare gains are the same for all agents). Our paper complements this literature, since we neither use a social welfare function nor assume ex-ante symmetry.

proximate sufficient-statistics formula depends only on observed nominal consumption expenditures and real exchange rates by country over time, the static input-output network at one point in time, and elasticities of substitution in consumption and production. In particular, applying the formula does not require modeling financial market imperfections, international ownership of assets, the productivity processes of each country, or separating consumption fluctuations due to wedges from those due to productivity shocks. We show, using Monte Carlo examples, that our second-order approximation performs well even for large shocks.

In Section 6, we take our sufficient-statistics deadweight-loss triangle formulas to microeconomic (household-level) and macroeconomic (country-level) data. In Section 6.1, we study misallocation from incomplete risk sharing using household consumption panel data from the United States (the Panel Study of Income Dynamics, PSID). We find that misallocation losses are similar to those in an off-the-shelf calibration of the Bewley model — roughly 20 percent of aggregate consumption in every period and state. Losses are greater the lower are the elasticity of intertemporal substitution and the interest rate.

In Section 6.2, we study misallocation from imperfect consumption smoothing and risk sharing using international macroeconomic data. For this second exercise, we assume each country has a representative agent, and quantify losses from the absence of complete financial markets between countries. We apply our international formula to a model with 32 countries, 54 industries in each country, and input-output linkages calibrated to the World Input-Output Database and macro data from 1970 to 2019. We find misallocation losses roughly in the 5 percent range. That is, if financial markets were complete and every household was compensated to be indifferent to the initial allocation, then there would be 5 percent of every consumption good left over in every date and state. Although we focus only on consumption wedges driven by financial frictions for consumers, we show that accounting for input-output linkages is quantitatively important.

The extent of misallocation crucially depends on the presence of heterogeneous growth rates across countries. If we exclude fast-growing countries in Asia (China, India, Indonesia, and South Korea), then misallocation losses drop to around 1 percent. Hence, the gains from completing financial markets are primarily due to unexploited gains from intertemporal trade between countries with different growth rates rather than risk sharing between similarly developed countries.

We conduct sensitivity analysis and show, once again, that losses are higher if the intertemporal elasticity of substitution is lower. We also show that losses are larger if the Armington trade elasticity is higher. Intuitively, there are more unexploited opportunities to share risk and smooth consumption, conditional on the data, if foreign and domestic goods are more substitutable.

These results underscore the potentially large welfare gains from more complete risk sharing, especially across individual households in domestic settings or between emerging and developed economies. Risk-sharing opportunities among countries at similar levels of development, on the other hand, are fairly limited.

Related literature. This paper is related to the literature that analyzes the efficiency implications of financial market incompleteness. There are two main branches of this literature. The first branch is concerned with domestic risk sharing of idiosyncratic household-level risks in closed-economy settings. The second branch analyzes efficiency of risk sharing in an international context with nontraded goods. We unify these literatures in the sense that the same misallocation measure and the same triangle logic apply in both domestic and international settings, and both can be implemented using limited, observable data. We discuss these two branches in sequence.

The first branch derives from Bewley (1972) and its extensions, including Imrohoroğlu (1989), Huggett (1993), and Aiyagari (1994). To evaluate aggregate welfare in this class of models, there are two common approaches. The first is to use a social welfare function, typically by appealing to the behind-the-veil-of-ignorance logic of Harsanyi (1955).[2] It is understood that social welfare functions, including the behind-the-veil one, embed some distributional judgment and require interpersonal comparisons. If preferences are risk averse, the behind-the-veil measure is averse to inequality. The second approach, following Benabou (2002) and then Floden (2001), aims to separate Pareto-efficiency considerations from distributional ones. These measures evaluate the value of an allocation using the sum of individual consumption certainty equivalents. Both the veil-of-ignorance approach and the Benabou (2002) approach rely on the assumption that households have the same preferences. This means they are not applicable to international settings where households in different countries consume different goods.

In the closed-economy setting, our measure agrees with the one proposed by Benabou (2002) if we abstract from labor-leisure choice. However, even abstracting from this, our paper complements Benabou (2002) and the literature that followed it by providing a second-order approximation of the efficiency losses from imperfect risk sharing. Our approximation formula, which is a Harberger (1964) triangles formula, requires only estimates of the elasticity of intertemporal substitution, the risk-free interest rate, and second moments of the distribution of household consumption allocations. This allows us to

[2] Some examples include Heathcote et al. (2008), Conesa et al. (2009), Dávila et al. (2012), Krueger et al. (2016), and Boar and Midrigan (2022).

quantify misallocation under weaker structural assumptions.

Our focus is on the distance of the allocation from the Pareto frontier. This means that, when financial markets are completed, we allow for lump-sum transfers between households to ensure everyone is compensated. Some papers in this literature, including Benabou (2002), consider constrained efficiency and second-best policies (with imperfect redistribution). Although our framework can be applied to study such questions, we do not pursue them in this paper (see Baqaee and Burstein, 2025b for an analysis of aggregate productivity with restricted redistributive instruments).[3]

The second branch of the risk-sharing literature focuses on the international dimension of the problem — typically taking the fact that some goods are nontraded into account. Examples include Van Wincoop (1994), Tesar (1995), Gourinchas and Jeanne (2006), Fitzgerald (2012), Heathcote and Perri (2014), Lewis and Liu (2015), Fitzgerald (2024), Corsetti et al. (2024), and Aguiar et al. (2025). The approach to quantifying inefficiency, or aggregate welfare, in this branch of the literature is more eclectic. Some papers assume ex-ante symmetric countries, so that the loss from restricting trade in financial assets is also symmetric. Some papers eschew aggregate comparisons and report country-by-country welfare changes only. Lastly, some papers use Bergson (1954)-Samuelson (1956) social welfare functions, typically a utilitarian function. Of course, there are also papers that analyze the efficiency properties of the decentralized equilibrium, without quantifying inefficiency per se, for example Cole and Obstfeld (1991), Backus and Smith (1993), and Lewis (1996).

Our paper also contributes to this literature. Given the way we measure misallocation (relative to complete markets), we do not need to impose that countries be ex ante symmetrical or to use a social welfare function. Accordingly, our measure relaxes the unrealistic assumption of symmetry, while avoiding interpersonal utility comparisons and remaining neutral on distributional issues. On a methodological front, we derive sufficient-statistics formulas that can be applied to the data without requiring knowledge of the productivity shocks that hit the economy.

Our paper is also related to recent work that provides approximate decompositions of changes in aggregate welfare using social welfare functions, for example Bhandari et al. (2021) and Dávila and Schaab (2022, 2023). Our paper differs from this literature because the measure of aggregate efficiency we use is distinct from the measures used in these

[3]This is related to Farhi and Werning (2012), who quantify the aggregate welfare gains from capital taxation in an incomplete-market model with private information, using the resources saved when implementing the inverse Euler equation while holding labor decisions and utilities unchanged. Another approach to evaluate second-best policies in incomplete-market environments is to look for robust Pareto improvements, which weakly relax all constraints, as in Aguiar et al. (2024).

papers in two ways. First, we do not specify a social welfare function. Second, we define aggregate efficiency exactly, and not as part of an approximate decomposition of aggregate welfare. This means that our measure can be integrated, allowing us to study the effect of large changes, and that, generically, it does not coincide with what is referred to as "efficiency" in these papers.

Of course, our paper is also related to a different literature that studies the efficiency consequences of misallocation, following Harberger (1954), and, more recently, Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). From a methodological and conceptual point of view, our paper is closely related to this literature, though we study a different type of misallocation. Whereas this literature typically emphasizes static cross-sectional misallocation in production, we focus on dynamic stochastic misallocation in consumption. Notwithstanding this difference, our methodological approach is similar. We analyze the distance to the Pareto-efficient frontier and use reduced-form wedges to capture the frictions in the decentralized equilibrium. Our contribution is to show that the same wedge-and-triangles apparatus used by Baqaee and Farhi (2020) to measure static production misallocation can be repurposed to measure dynamic, stochastic misallocation in consumption due to incomplete markets.[4]

Finally, this paper has two companions: Baqaee and Burstein (2025b) and Baqaee and Burstein (2025a). The approach in this paper is a specific application of the general framework for measuring misallocation and aggregate productivity developed in Baqaee and Burstein (2025b). In the other companion paper, Baqaee and Burstein (2025a), we apply the same framework to study changes in aggregate productivity in spatial economies with discrete choice and heterogeneous consumer tastes (rather than incomplete financial markets and continuous choice, as in this paper).

2 Misallocation Due to Market Incompleteness

Consider an economy populated by agents indexed by h. Each agent h has non-satiated ordinal preferences $\succeq_h$ over time- and state-contingent commodity bundles $\boldsymbol{c}_h$. The consumption stream $\boldsymbol{c}_h$ contains everything over which the agent has preferences — so, in a model with labor-leisure choice, it includes leisure. Preferences are represented by utility functions $u_h(\boldsymbol{c}_h)$. A *consumption allocation* is a collection $\boldsymbol{c} = \{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_H\}$ of consumption bundles for each agent.

[4]Some papers, such as Buera et al. (2011), Midrigan and Xu (2014), Moll (2014), and Bigio and La'O (2016), study misallocation from financial frictions on firms. It would be interesting, but beyond the scope of this paper, to combine those frictions with the ones on households that we focus on. Proposition 1 can be applied to do this.

Denote the initial equilibrium consumption allocation by $\boldsymbol{c}^0$. This is the allocation that we observe in the data, and features imperfect risk sharing and consumption smoothing due to market incompleteness. We measure misallocation by the distance of $\boldsymbol{c}^0$ from the frontier of technologically feasible consumption allocations. Following Debreu (1951), we do this by proportionally contracting total factor productivity until it is possible to make every household just indifferent to the initial equilibrium allocation. The contraction factor measures the degree of economic waste imposed by market incompleteness in terms of foregone total factor productivity.

More formally, let $\mathcal{C}(Z)$ be the set of technologically feasible consumption allocations given a factor-augmenting productivity shifter Z (common to all primary factors of production). Since non-constant returns to scale can always be captured by adding additional primary factors, we can assume, without loss of generality, that $\mathcal{C}(Z)$ is homogeneous of degree one in Z. That is, doubling Z also doubles the set of feasible consumption allocations. We normalize $Z = 1$ in the initial equilibrium, so that the economy's technologically feasible consumption possibility set is $\mathcal{C}(1)$. By definition, $\boldsymbol{c}^0 \in \mathcal{C}(1)$.

Definition 1 (Distance to Frontier)**.** *Misallocation* is measured by the maximum contraction of total factor productivity such that every agent can be kept at least indifferent to the incomplete markets allocation. Formally,

$$A \equiv \max \left\{ Z > 0 : \text{there is } \boldsymbol{c} \in \mathcal{C}(1/Z) \text{ with } u_h(\boldsymbol{c}_h) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h \right\}. \tag{1}$$

Since $\boldsymbol{c}^0 \in \mathcal{C}(1)$, misallocation A is always weakly greater than one, and in general, the fraction of every factor that can be discarded is $1 - 1/A$. If $A = 1$, then this implies that $\boldsymbol{c}^0$ is Pareto efficient. If A is greater than one, say $A = 1.01$, then it is possible to make everyone at least as well off as in the initial equilibrium and discard around 1 percent of every primary factor (or equivalently, of every good). Thus, A is a measure of the economic waste due to incomplete markets. Importantly, in calculating A, we do not need to take a stance on which agents would or should receive these extra resources if one were to complete markets. That is, this aggregate efficiency measure is silent on redistribution. Crucially, and unlike a social welfare function, A is invariant to increasing monotone transformations of utility functions.[5]

To calculate A, we first define for each agent the consumption-equivalent variation as in Lucas (2003).

[5]In Appendix B we show that, if all household have common homothetic preferences (as in Bewley-Aiyagari models), then A can also be interpreted in terms of the sum of compensating variations from completing financial markets.

Definition 2. The *consumption-equivalent variation* for agent h, denoted by $\tilde{u}_h(\boldsymbol{c}_h)$, is the solution to

$$u_h\left(\frac{\boldsymbol{c}_h}{\tilde{u}_h}\right) = u_h(\boldsymbol{c}_h^0).$$

In words, $\tilde{u}_h(\boldsymbol{c}_h)$ is the factor by which the consumption stream $\boldsymbol{c}_h$ has to be scaled to make it exactly as desirable to agent h as $\boldsymbol{c}_h^0$. By construction, $\tilde{u}_h(\boldsymbol{c}_h)$ is homogeneous of degree one in $\boldsymbol{c}_h$ and invariant to monotone transformations of utility.

The next proposition shows that A can be computed as the largest common consumption-equivalent variation that can be delivered to all households given the set of feasible allocations under complete markets, $\mathcal{C}(1)$.

Proposition 1 (Calculating misallocation)**.** *The misallocation measure A can equivalently be written as*

$$A = \max_{a,\boldsymbol{c}} \{a : \tilde{u}_h(\boldsymbol{c}_h) \geq a \text{ for every } h \text{ and } \boldsymbol{c} \in \mathcal{C}(1)\}. \tag{2}$$

Denote the allocation that solves (2) by $\boldsymbol{c}^*$. Then, $\boldsymbol{c}^*/A$ is the allocation that solves the original problem in equation (1). The allocation $\boldsymbol{c}^*$ is on the Pareto frontier but there is no sense in which it is the "socially-optimal allocation" if we were to remove misallocation. Rather, it is simply an analytical device for measuring misallocation A. Indeed, we never define *the* optimal allocation on the consumption possibility set $\mathcal{C}(1)$. That is, if financial markets were completed, and the entire efficient frontier of $\mathcal{C}(1)$ were available, we do not take a stance on which point on that frontier is optimal.

We use Proposition 1 throughout the paper to study misallocation due to incomplete financial markets both in closed and open economies.

3 Baseline Closed Economy

We begin with a simple economy where all agents consume the same consumption good every period but they do not share risks efficiently. This nests one-sector models like Bewley (1972) and Huggett (1993), but it also accommodates multi-sector versions of these models with input-output linkages. We begin this section by setting up the environment. We then provide exact and approximate characterizations of misallocation. In this section, we abstract from labor-leisure choice, capital accumulation, and nontraded goods (i.e., differences in consumption baskets). We extend our results to allow for these additional ingredients in subsequent sections.

3.1 Environment

Each household, indexed by h, has intertemporal preferences over state-contingent consumption streams $\boldsymbol{c}_h$ represented by a common utility function

$$u(\boldsymbol{c}_h) = \frac{1}{1-1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}. \tag{3}$$

Here, $c_{ht}(s)$ denotes the composite consumption of household h at time t in state s, the discount factor is $\beta < 1$, and $\eta > 0$ is the elasticity of intertemporal substitution (EIS). The probability of state s is denoted by $\pi(s)$, where each state s indexes a sample path of shocks.

In every period t of every state s, production takes place. We assume that production in each period is statically efficient.[6] Since we abstract from capital accumulation and endogenous labor choice, and all households value the same consumption good, we do not need to specify the exact nature of the production structure.

There may be many producers, heterogeneous goods, input-output linkages, and arbitrary production functions, with arbitrary returns to scale and any pattern of exogenous technological shocks. Since all households value the same consumption good, and the production side of the economy is efficient, the equilibrium allocation always maximizes aggregate consumption in every date and state. Hence, we need only to specify the aggregate quantity of the consumption good in period t and state s, $y_t(s)$, and note that it remains unchanged if we complete financial markets. This is because completing markets only affects the distribution of this good across households in each date and state, not its total quantity. We denote exogenous productivity shifters, which affect the total quantity of the consumption good, by $z_t(s)$. We do not need to be explicit about the exact nature of these shocks, but their presence means that $y_t(s)$ is not constant across dates and states.

The resource constraint for the consumption good in date t and state s is therefore

$$\sum_h c_{ht}(s) = y_t(s).$$

The initial equilibrium allocation, $\boldsymbol{c}^0$, satisfies these resource constraints. Proposition 1

[6]Production is neoclassical and all producers set price equal to marginal cost. This implies that the allocation is statically efficient: holding fixed consumption allocations in every other period and state, and focusing only on a single period and state, it is not possible to make one agent better off without making someone worse off.

implies that

$$A = \max_{a,\boldsymbol{c}} \left\{ a : \tilde{u}_h(\boldsymbol{c}_h) = a \text{ for every } h, \text{ and } \sum_h c_{ht}(s) = y_t(s) \text{ for every } t \text{ and } s \right\}. \quad (4)$$

In the next section we characterize the solution to this problem.

3.2 Exact Characterization

We can characterize A using the certainty-equivalent function. Define the certainty-equivalent variation of a consumption process $\boldsymbol{c}_h$ to be the function $CE(\boldsymbol{c}_h)$ that solves

$$u(\boldsymbol{c}_h) = u(\mathbf{1}\, CE(\boldsymbol{c}_h)).$$

In words, $CE(\boldsymbol{c}_h)$ is the amount of consumption the agent needs in every date and state to be indifferent to the consumption stream $\boldsymbol{c}_h$. In closed form,

$$CE(\boldsymbol{c}_h) = \left[(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{ht}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $\mathbb{E}_0$ denotes the time-zero expectation over states. Note that, if preferences are homothetic, the consumption-equivalent variation evaluated at $\boldsymbol{c}_h$, $\tilde{u}_h(\boldsymbol{c}_h)$, is equal to the ratio of the certainty-equivalents of $\boldsymbol{c}_h$ relative to the initial distorted allocation:

$$\tilde{u}_h(\boldsymbol{c}_h) = \frac{CE(\boldsymbol{c}_h)}{CE(\boldsymbol{c}_h^0)}.$$

To solve for A, observe that any Pareto-efficient consumption allocation must satisfy

$$c_{ht}(s) = \alpha_h y_t(s)$$

for some household-specific $\alpha_h \geq 0$ with $\sum_h \alpha_h = 1$.[7] By Equation (4) and homogeneity of degree one of the certainty-equivalent function,

$$A = \tilde{u}_h(\boldsymbol{c}_h) = \frac{\alpha_h\, CE(\boldsymbol{y})}{CE(\boldsymbol{c}_h^0)} \qquad \text{for every } h,$$

where $\boldsymbol{y}$ is the stream of aggregate consumption, $\{y_t(s)\}_{t,s}$. Solving for α_h and setting

[7] We only need to consider Pareto-efficient allocations in the consumption possibility set $\mathcal{C}(1)$.

$\sum_h \alpha_h = 1$ gives the following characterization of misallocation in terms of certainty-equivalents.[8]

Proposition 2 (Misallocation in economies with a common consumption good)**.** *Misallocation, defined by* (4)*, is given by*

$$A = \frac{CE(\sum_h \boldsymbol{c}_h^0)}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{5}$$

In words, A is the certainty-equivalent of the aggregate consumption process relative to the sum of households' certainty-equivalents. The intuition is simplest in the special case in which aggregate output is constant across dates and states. In that case, A is the ratio of aggregate output to the minimum constant level of consumption that leaves every household indifferent to the equilibrium with incomplete markets, and a fraction $1 - 1/A$ of output is left over after compensating everyone. When aggregate output fluctuates across dates or states, it is no longer possible to compensate everyone by keeping their consumption constant. As a result, more resources are needed to keep everyone indifferent, and the amount left over after compensation is smaller.

Equation (5) is similar to the definition of aggregate efficiency provided by Benabou (2002), except here this expression is a result — given our general notion of misallocation — rather than a definition. The fact that we have a general definition of misallocation makes it straightforward to extend (5) to more complex environments with labor-leisure, capital accumulation, and differences in consumption baskets across agents. When we consider these ingredients in the following sections, our measure of misallocation A differs from the Benabou (2002) measure.

Contrast to the veil-of-ignorance. We contrast A to a popular measure of aggregate welfare in the literature: the veil-of-ignorance social welfare function.[9] Whereas A is a measure of misallocation, the veil-of-ignorance measure combines efficiency with distributional concerns.

Define the veil-of-ignorance certainty-equivalent of a consumption allocation $\boldsymbol{c}$ by:

$$u(\mathbf{1}CE^{VOI}) = \sum_h \frac{1}{H} u(\boldsymbol{c}_h).$$

[8] To obtain (5), we do not need CRRA preferences. It is enough that all households have common, concave, and homothetic preferences.

[9] Since all households in the baseline model have common preferences, this measure is unambiguous to define. See Eden (2020) for a detailed discussion of the veil-of-ignorance approach to quantifying social welfare, and how it must be adapted in the presence of heterogeneous preferences. In particular, when agents have heterogeneous preferences, one cannot just naively sum up cardinal utility functions and think of it as the veil-of-ignorance measure.

In words, CE^{VOI} is the certainty-equivalent of a population-weighted lottery of the consumption allocation of each agent. We can calculate the ratio of the value of first-best, according to CE^{VOI}, relative to the value of the incomplete-markets allocation:

$$A^{VOI} = \frac{\max_{\boldsymbol{c}\in\mathcal{C}(1)} CE^{VOI}(\boldsymbol{c})}{CE^{VOI}(\boldsymbol{c}^0)} = \frac{CE(\sum_h \boldsymbol{c}_h^0)}{\left(\sum_h \left(CE(\boldsymbol{c}_h^0)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}, \tag{6}$$

where the numerator uses the fact that the first-best allocation with this social welfare function would split aggregate consumption uniformly across all agents. (See Appendix A for a derivation). Comparing (5) to (6), we see that the veil-of-ignorance measure, A^{VOI}, uses risk-preferences to discipline inequality-aversion, whereas A does not feature this effect.[10,11]

To see that A^{VOI} is affected by distributional concerns, consider an example where every household consumes a constant fraction, α_h, of aggregate consumption. This allocation is Pareto efficient; accordingly, $A = 1$ and there is no misallocation.[12] However, as there is inequality in the initial equilibrium (α_h varies across households), $A^{VOI} > 1$ (unless $\eta = \infty$ and households are risk-neutral).

3.3 Approximate Characterization

We now provide a second-order approximation of misallocation. This approximation serves two purposes. First, it provides intuition for how parameters affect misallocation. Second, and more importantly, it identifies approximate sufficient statistics that can be taken to the data without requiring complete knowledge of the entire joint distribution of consumption allocations and productivity shifters.

To derive this approximation, we introduce the concept of a general equilibrium with wedges. We decentralize the distorted incomplete-markets allocation using household-

[10]In other words, the veil-of-ignorance measures sets the Atkinson (1970) parameter for inequality-aversion equal to the coefficient of relative risk aversion.

[11]In this example, the veil of ignorance can also be thought of as a utilitarian social welfare function (sum of utilities) with a particular cardinalization of the utility function. However, with other cardinalizations of the same preferences, the sum of utilities will have different implications. This is because "the" utilitarian welfare function is not well-defined as it depends on how each utility function is cardinalized. For example, if instead of using the functional-form of u, defined above, we cardinalize the same preferences using the monotone nonlinear transformation $(u(\boldsymbol{c}_h)(1-1/\eta))^{\frac{\eta}{\eta-1}} = \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{ht}^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$, then the utilitarian social welfare function would have zero inequality aversion. In contrast, our measure of misallocation, A, is invariant to monotone transformations of utility.

[12]To see this, note that $CE(\boldsymbol{c}_h^0) = \alpha_h CE(\sum_{h'} \boldsymbol{c}_{h'}^0)$ and, using Proposition 2, it follows that $A = 1$.

specific, state- and date-contingent consumption taxes. Let $\mu_{ht}(s)$ denote the *wedge*, which is an implicit tax, on the consumption of household h at time t in state s. In the decentralization with wedges, the intertemporal budget constraint for household h is

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)p_t(s)c_{ht}(s) \leq I_h,$$

where $q_t(s)$ is the price of an Arrow security, $p_t(s)$ is the price of the consumption good in state s at time t not including the wedge, and I_h is initial wealth.

We now define general equilibrium with wedges.

Definition 3 (Equilibrium with Wedges)**.** A general equilibrium with wedges is a collection of prices and quantities such that: (1) each household chooses consumption to maximize utility taking prices, consumption tax wedges, and wealth as given; (2) every producer chooses inputs to minimize costs, taking input prices as given, and sets its price equal to marginal cost; (3) resource constraints are satisfied; (4) wedge revenues are rebated to households lump-sum.

The following proposition shows that the initial equilibrium consumption allocation, $\boldsymbol{c}^0$, can be decentralized using some pattern of household-state-date-specific consumption wedges.

Proposition 3 (Decentralization with Wedges)**.** *Consider an initial equilibrium allocation $\boldsymbol{c}^0$. Let $\omega_{ht}(s) = c^0_{ht}(s)/\sum_{h'} c^0_{h't}(s)$ be household h's share of consumption in state s at date t. Then setting*

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \omega_{ht}(s) - \log \omega_{h0}\right] \tag{7}$$

implies that $\boldsymbol{c}^0$ is a general equilibrium with those wedges. This equilibrium is supported by some lump-sum transfers across households.

We do not need to specify the lump-sum transfers explicitly. For our purposes, all that matters is that there exists an equilibrium with the wedges in (7), with appropriate transfers, that supports $\boldsymbol{c}^0$ as an equilibrium allocation. Intuitively, these wedges summarize all risk-sharing imperfections as price distortions. These wedges depend only on consumption shares and the EIS, not on the details of income or asset positions.[13]

[13] The wedges in Equation (7) are distinct from the wedges in Berger et al. (2023). They consider preference shifters that, in a representative agent economy, replicate the path of aggregate outcomes (e.g., aggregate consumption, hours, etc.) from a heterogeneous agent New Keynesian model. They show that deviations from perfect risk sharing map onto discount factor shocks in the representative agent model. They then consider the reduction in output volatility in the absence of these as-if discount factor shocks. In contrast,

Baqaee and Burstein (2025b) show that misallocation in heterogeneous-agent models can be approximated using deadweight-loss triangles. Applying Proposition 4 in that paper to the environment considered here, we can express misallocation to a second-order approximation as

$$\log A \approx \frac{1}{2}\,\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\sum_h \omega_{h0}\left(\log c^*_{ht}(s)-\log c^0_{ht}(s)\right)\,\log\mu_{ht}(s)\right], \tag{8}$$

where $c^*_{ht}(s)$ is the consumption allocation that solves the problem in (4), r is the one-period risk-free rate, and ω_{h0} is the expenditure share of household h in the initial period. The approximation error is stated in Proposition 4.

The intuition is the same as the traditional deadweight-loss (Harberger) triangle logic. The height of the triangle is measured by the wedge from (7). The base of the triangle is the change in consumption caused by removing all wedges. The area of the triangle is one half the product of the base and the height. We sum up deadweight loss triangles over households using net-present-value expenditure weights $\omega_{h0}r/(1+r)^{t+1}$. Triangles for richer households receive larger weights because of the compensation principle. Eliminating distortions for richer households is more valuable because a rich household is then able to compensate everyone else more easily, and this results in more surplus.

Substituting into (8) the formulas for the wedges and the implied consumption changes from removing them, the following proposition expresses misallocation in terms of the cross-sectional average of the total lifetime variance of log consumption shares for individuals.

Proposition 4 (Approximate Misallocation for Baseline)**.** *To a second-order approximation, misallocation, defined by* (4), *is*

$$\log A \approx \frac{1}{2}\frac{1}{\eta}\sum_h \omega_{h0}\,\mathrm{Var}_{r,\pi}[\log\omega_{ht}(s)\mid h], \tag{9}$$

where $\mathrm{Var}_{r,\pi}$ *denotes the variance of* $\log\omega_{ht}(s)$ *for household h using* $\frac{r}{(1+r)^{t+1}}\pi(s)$ *as weights. The approximation error is of order* $(\log\mu)^3$ *and* $(\log\mu)^2\Delta\log z$, *where* $\Delta\log z$ *denotes deviations of technology parameters from some constant value.*[14]

the wedges in Proposition 3 replicate a microeconomic, rather than just aggregate, allocation in a heterogeneous agent general equilibrium with wedges. We use these wedges to construct a deadweight loss triangle formula for incomplete-market models.

[14]Technically, because wedges could be endogenous, the approximations are taken in some limit of an exogenous parameter that causes all the log wedges, $\log\mu$, to go to zero. Typically, this parameter is idiosyncratic risk. As idiosyncratic risk goes to zero, risk-sharing wedges $\log\mu$ go to zero.

The advantage of Proposition 4 is that it can be readily taken to consumption panel data without solving a fully-specified incomplete-market model. Given consumption data, misallocation depends on the risk free rate r and the EIS η. A higher EIS makes fluctuations in consumption less costly and therefore decreases misallocation. In addition, as the risk-free rate goes to infinity (i.e., households become fully myopic) misallocation goes to zero because intertemporal distortions have no welfare consequences. Finally, note that permanent differences in consumption shares across households do not cause any misallocation.

3.4 Quantitative Example

We use Propositions 2 and 4 to measure misallocation from market incompleteness using off-the-shelf calibration of Bewley (1972). We use this quantitative example to show how the costs of market incompleteness change as a function of parameters like idiosyncratic risk, borrowing constraints, and public debt. We also use this example to test the performance of our second-order approximation and its finite sample properties.

Model. There is a unit mass of households, indexed by $h \in [0,1]$, with preferences as in (3) subject to a per-period budget constraint

$$c_{ht} + a_{ht+1} = (1-\tau)z_{ht} + (1+r_t)a_{ht},$$

where a_{ht} is the quantity of a risk-free bond held by h, z_{ht} is risky labor productivity, and τ is the tax rate. Each household faces a borrowing constraint

$$a_{ht+1} \geq -\underline{a}.$$

Productivity evolves according to

$$\log z_{ht} = \rho \log z_{ht-1} + \sigma \epsilon_{ht},$$

where ϵ_{ht} is an i.i.d standard normal random variable. The government has issued B risk-free bonds and runs a balanced budget every period using labor income taxes, so that

$$rB = \tau.$$

Market clearing condition for goods and bonds is

$$\int_0^1 c_{ht} dh = \int_0^1 z_{ht} dh = y = 1, \qquad \text{and} \qquad \int_0^1 a_{ht} dh = B.$$

Parameterization. We use a quarterly calibration. We set quarterly persistence of log income to be $\rho = 0.975$ with standard deviation 0.16 to match estimates of the quarterly persistence and the cross-sectional standard deviation of the persistent component of log income in the United States.[15] We set the borrowing limit to be -5, so households can borrow at most five times their quarterly income. We set the annual risk-free rate $r = 5\%$ and the EIS $\eta = 0.5$. Finally, we set $B = 5.6$ — so that total bonds outstanding relative to quarterly output is 560% (or 140% of annual GDP).

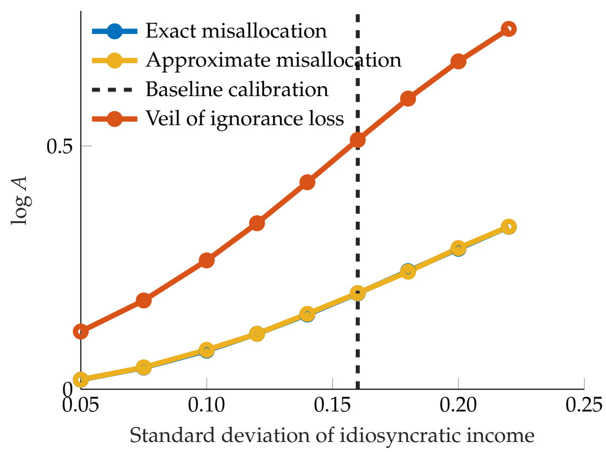


Figure 1: Losses as a function of idiosyncratic income risk. Dashed line is benchmark. Exact and approximate misallocation are visually identical.

Results. Figure 1 plots the extent of misallocation, calculated using Proposition 2, and compares it to the second-order approximation from Proposition 4, using the steady-state invariant distribution as the initial equilibrium. The approximation performs well and, as expected, becomes exact as $\sigma \to 0$. The benchmark calibration is indicated by the dashed black line, where misallocation is $\log A \approx 0.20$. This means that if agents perfectly insure each other and everyone is kept indifferent to their status quo allocation, then there is 20 log points (or $1 - \exp(-0.20) \approx 18$ percent) of output left over to be split across agents as desired.

[15]See Rognlie (2024). We target a cross-sectional standard deviation of log income equal to 0.7, which means that the standard deviation of the innovations must be $\sigma = 0.7 \times \sqrt{1 - 0.975^2}$.

We also compare the equilibrium allocation to the first-best allocation under the veil-of-ignorance criterion, using equation (6). The veil-of-ignorance measure, which penalizes inequality across agents analogously to uncertainty for each agent, assigns more than double the losses to the status quo. That is, behind the veil, households would be prepared to give up 51 log points ($\approx$ 40 percent) of aggregate consumption if they could equalize consumption across dates, states, and the cross-sectional population relative to being given the population-weighted lottery of consumption allocations.

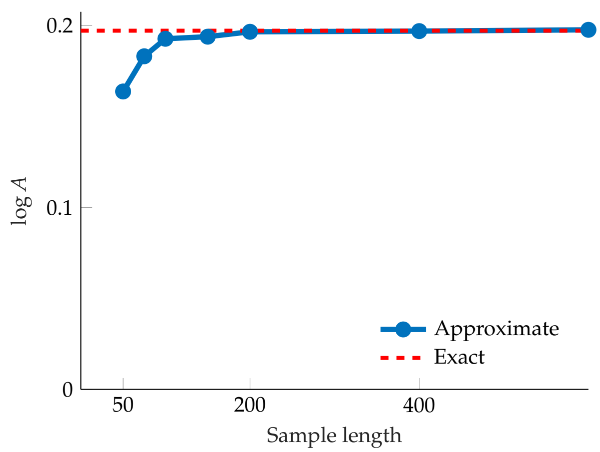


Figure 2: Quality of approximation for benchmark calibration

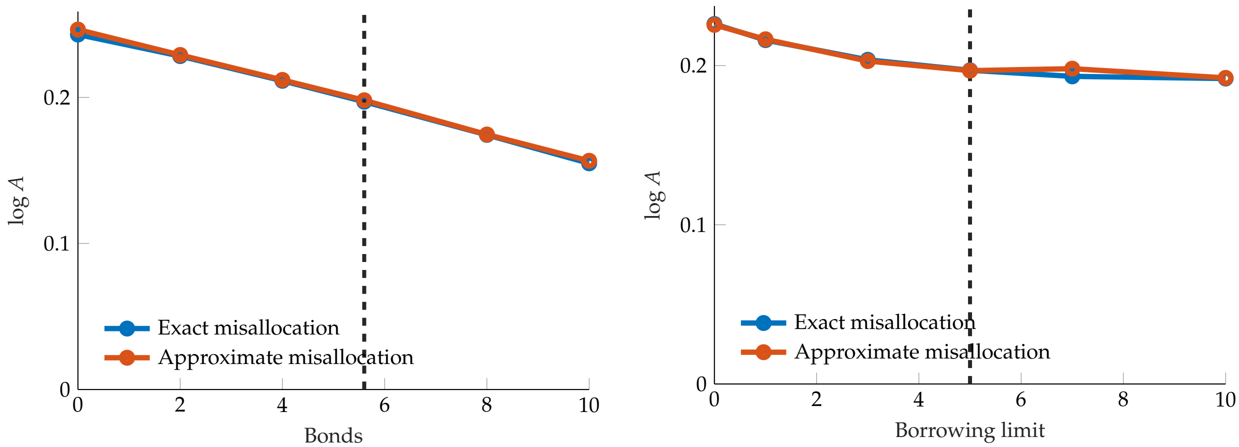
Figure 1 shows that the second-order approximation is extremely accurate. Figure 2 plots the quality of the second-order approximation against the exact misallocation losses as the sample length used in the approximation increases. The second-order approximation stabilizes after about 100 quarters (25 years), but suffers from some small-sample bias when the number of quarters is significantly shorter than that. With a truncated sample, the second-order approximation underestimates the extent of misallocation because the Harberger triangles in the first period are, by construction, equal to zero.[16]

Figure 3 plots misallocation in the invariant distribution as a function of the aggregate supply of bonds and the borrowing limit. Misallocation falls mildly as the supply of bonds or borrowing limit is increased. The approximation continues to perform well.

4 Extensions with Leisure and Capital

In this section, we quantify misallocation allowing for endogenous labor-leisure choice and capital accumulation.

[16]The second-order approximation is much less sensitive to the number of households in the sample.



(a) Bond supply

(b) Borrowing limit

Figure 3: Misallocation as a function of parameters (dashed line is the benchmark calibration).

4.1 Extension with Labor-Leisure Choice

Suppose households have preferences over consumption goods and leisure:

$$u(\boldsymbol{c}_h, \boldsymbol{l}_h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(c_{ht}(s), l_{ht}(s)).$$

Each household has a unit endowment of time, which they devote either to leisure, $l_{ht}(s)$, or to work, $1 - l_{ht}(s)$. For simplicity, assume production converts labor into the consumption good linearly. Hence, the resource constraint for consumption at date t in state s is

$$\sum_h c_{ht}(s) = y_t(s) = \sum_h z_{ht}(s) \left(1 - l_{ht}(s)\right), \tag{10}$$

where $z_{ht}(s)$ is the idiosyncratic productivity of household h in date t and state s. In the baseline model, leisure $l_{ht}(s)$ is normalized to be zero. The consumption possibility set, denoted by $\mathcal{C}(1)$, now consists of all consumption and leisure processes that are consistent with resource constraints. Elements of $\mathcal{C}(1)$ are consumption and leisure processes for each household, where each agent's leisure process must be in the unit interval, $l_{ht}(s) \in [0, 1]$, and consumption processes must satisfy (10).

As before, A is the fraction by which we can shrink aggregate total factor productivity, or equivalently $\mathcal{C}(1)$, and still keep every agent at least indifferent to the initial equilibrium. The scalar $1 - 1/A$ measures how much of every good, including leisure, in every state and date is left over after every agent has been made indifferent. Equivalently, this is a measure of the amount of the time endowment that is being wasted (not used for labor or leisure) due to market incompleteness.

Proposition 1 implies that

$$A = \max_{a,\boldsymbol{c},\boldsymbol{l}} \left\{ a : \tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h) = a \text{ for every } h, l_{ht}(s) \in [0,1], \text{ and (10) holds} \right\}.$$

where consumption-equivalents, $\tilde{u}_h$, now include leisure, which is treated like any other consumption good. Namely, $\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h)$ is implicity defined by the equation

$$u(\boldsymbol{c}_h^0, \boldsymbol{l}_h^0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v\left(\frac{c_{ht}(s)}{\tilde{u}_h}, \frac{l_{ht}(s)}{\tilde{u}_h}\right). \tag{11}$$

We provide an example of $\tilde{u}_h$ with leisure below.

Example 1 (Balanced-growth preferences). Suppose the intratemporal utility function, v, is

$$v(c_h, l_h) = \frac{1}{1-1/\eta}\left[c_h^{\gamma} l_h^{1-\gamma}\right]^{1-1/\eta}. \tag{12}$$

Then $\tilde{u}_h$ is

$$\tilde{u}_h(\boldsymbol{c}_h, \boldsymbol{l}_h) = \left[\frac{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[c_{ht}(s)^{\gamma} l_{ht}(s)^{1-\gamma}\right]^{1-1/\eta}}{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[c_{ht}^0(s)^{\gamma} l_{ht}^0(s)^{1-\gamma}\right]^{1-1/\eta}}\right]^{\frac{1}{1-1/\eta}}.$$

Comparison with Benabou (2002) measure of aggregate efficiency. As mentioned earlier, our definition of efficiency is different from a frequently used alternative in the literature, such as Benabou (2002), Floden (2001) and Boar and Midrigan (2022). To compare two allocations, the literature following Benabou (2002) defines the consumption-equivalent of a consumption process $\boldsymbol{c}_h$ and a leisure process $\boldsymbol{l}_h$ to be the function $CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$ that solves

$$u(\mathbf{1}CE, \mathbf{1}\bar{l}) = u(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

where $\mathbf{1}CE$ and $\mathbf{1}\bar{l}$ are constant streams over dates and states, and $\bar{l}$ is some constant level of leisure (e.g., average leisure). The efficiency of an allocation is defined by $\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$.

One way to see the difference between A and this measure is to note that the Benabou (2002) measure assigns different values to different allocations on the Pareto frontier. Suppose that preferences take the standard functional form in Example 1. In this case, this measure of efficiency can be written as

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \left(u(\boldsymbol{c}_h, \boldsymbol{l}_h)\right)^{\frac{\eta}{(\eta-1)\gamma}},$$

where the constant depends on β, η, γ, and $\bar{l}$. Consider the simple case where labor productivity is equal to one in every date and state for every agent. Then we can show that for every allocation $(\boldsymbol{c}, \boldsymbol{l})$ on the Pareto frontier, we can write

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \alpha_h^{\frac{1}{\gamma}}, \tag{13}$$

for some numbers on the unit simplex, $\{\boldsymbol{\alpha} \geq 0 : \sum_h \alpha_h = 1\}$. (See Appendix A for a derivation). In this expression, $\boldsymbol{\alpha}$ are coordinates of the Pareto frontier — i.e. they can be interpreted like Pareto weights — the higher is α_h for household h, the higher is the utility of that agent. Clearly, unless $\gamma = 1$, and there is no labor-leisure choice, (13) assigns different values to different points on the Pareto-frontier. Surprisingly, this measure assigns (weakly) higher values to more unequal Pareto weights since $\gamma \leq 1$. Indeed, by continuity, this shows that there are Pareto-inefficient allocations that receive a higher value according to this measure than alternative Pareto-efficient allocations with less inequality. Therefore, once we have labor-leisure choice, this measure is not neutral with respect to pure redistributions (and indeed, prefers inequality). In contrast, our measure detects zero misallocation (i.e. $A = 1$) for any allocation on the Pareto-efficient frontier.

Quantitative example with labor-leisure choice. Assume intratemporal preferences take the MaCurdy (1981) form

$$v(c_h, l_h) = \frac{1}{1 - 1/\eta} c_h^{1-\frac{1}{\eta}} + \phi_0 l_h^{1-\frac{1}{\phi}}. \tag{14}$$

We set the EIS equal $\eta = 0.5$, $\phi = 0.5$, and calibrate ϕ_0 so that leisure is, on average, equal to 40 percent of the time endowment (the implied Frisch elasticity is roughly $1/3$). We recalibrate the discount factor and the standard deviation of the productivity shocks to hit the same interest rate $r = 0.05$ and the cross-sectional variance of consumption as in the baseline calibration above. The remaining parameters are the same as in the calibration in Section 3.4.

The losses are shown in Figure 4 as a function of idiosyncratic risk σ calculated using Proposition 1. Even as σ goes to zero, the losses are non-zero since there is a tax on labor. Our definition of A captures all wedges that prevent the economy from reaching the Pareto frontier, not just incomplete markets; in this calibration, the labor income tax generates a residual misallocation even in the absence of idiosyncratic risk. However,

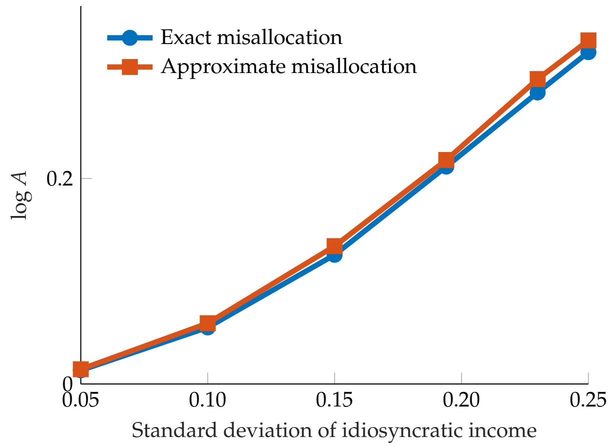


Figure 4: Misallocation with labor-leisure choice as a function of idiosyncratic income risk. Dashed line indicates the benchmark value of σ.

this effect is small because the baseline tax rate is small.[17] Misallocation for the baseline parameters is 21 log points, which is very similar to the baseline model. Furthermore, the second-order approximation in Proposition 4 continues to perform very well, even though it is derived for a model without labor-leisure choice.[18]

4.2 Extension with Capital Accumulation

We now consider an extension with capital accumulation along the lines of Aiyagari (1994). For simplicity, assume labor is inelastically supplied, but capital is not. Aggregate output in each period and state is

$$y_t(s) = z_t(s)k_t(s)^{\alpha}, \tag{15}$$

where $z_t(s)$ is a technology shifter, and we impose that the aggregate endowment of labor is equal to one. Capital accumulation satisfies

$$k_{t+1}(s) = (1-\delta)k_t(s) + x_t(s), \tag{16}$$

[17] Baqaee and Burstein (2025b) discuss how to measure misallocation relative to a constrained efficient frontier. The approach in that paper can be used to study misallocation relative to a distorted frontier where financial markets are completed but labor income taxes are held fixed.

[18] The second-order approximation with labor-leisure choice is very similar to the expression in Proposition 4, with an extra term capturing the fact that the consumption-to-leisure choice is distorted by a labor income tax. This term is quantitatively small because the baseline tax rate is small and the Frisch elasticity of labor supply is low.

where $x_t(s)$ is investment. Denote the initial capital stock by k_0. The aggregate resource constraint for output is

$$y_t(s) = c_t(s) + x_t(s) = \sum_h c_{ht}(s) + x_t(s). \tag{17}$$

In this dynamic economy, the primary factor endowments are labor, in every date and state, and the initial capital stock. Capital stocks after the initial date are not primary factors since since they are produced endogenously from initial capital and labor. If we scale the endowment of labor in every date and state, along with the initial capital stock, then the consumption possibility set expands one-for-one. We measure misallocation as the fraction of primary factors that are wasted.

Proposition 1 continues to hold with capital accumulation. Specifically, A solves

$$A = \max_{\boldsymbol{c},\boldsymbol{x}} \left(\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} \right)^{\frac{\eta}{\eta-1}},$$

subject to (15), (16), (17), and $\frac{u(\boldsymbol{c}_h)}{u(\boldsymbol{c}_h^0)} = \frac{u(\boldsymbol{c}_{h'})}{u(\boldsymbol{c}_{h'}^0)}$ for every $h' \in H$ and some initial capital stock k_0. Here, we use the fact that $\tilde{u}_h(\boldsymbol{c}_h) = (u(\boldsymbol{c}_h)/u(\boldsymbol{c}_h^0))^{\eta/(\eta-1)}$ in this environment, which in the solution to Proposition 1, must be equalized across h. This implies the following.

Proposition 5 (Misallocation with capital accumulation)**.** *Let $c_t^*(s)$ be the optimal (aggregate) consumption choice in period t and state s of a representative agent in the neoclassical growth model with initial capital stock k_0.*[19] *Then*

$$A = \frac{CE(\{c_t^*(s)\}_{t,s})}{\sum_h CE(\boldsymbol{c}_h^0)}. \tag{18}$$

That is, A is equal to the ratio of the certainty-equivalent of the aggregate consumption process from a neoclassical growth model given the initial aggregate capital stock k_0 relative to the sum of the certainty-equivalent of each agent in the initial equilibrium. Compared to Proposition 2, this means calculating misallocation now has one more step: solving for the transition dynamics in a standard neoclassical growth model given initial capital stock k_0. This additional step is needed because misallocation now involves not

[19] In particular, $c_t^*(s)$ solves an aggregate Euler equation $(c_t^*)^{-\frac{1}{\eta}} = \beta \mathbb{E}_t \left[\left(\alpha z_{t+1} k_{t+1}^{\alpha-1} + 1 - \delta\right) \left(c_{t+1}^*\right)^{-\frac{1}{\eta}} \right]$ and satisfies aggregate resource constraints $c_t^* = z_t k_t^\alpha + (1-\delta)k_t - k_{t+1}$, given some initial k_0 and the standard transversality condition. The vector $\{c_t^*(s)\}_{t,s}$ is the stream of aggregate consumption across states and dates.

only imperfect risk-sharing but also over-accumulation of capital relative to the Pareto-efficient allocation, where the efficient capital stock can be obtained from a representative-agent neoclassical growth model.

Quantitative example with capital accumulation. Proposition 5 does not fully specify households' portfolio problems, since misallocation only depends on the initial consumption allocation, not on the specific microfoundation for how that consumption allocation came about. To illustrate Proposition 5 quantitatively, we assume that the initial consumption allocation is the equilibrium of a standard Aiyagari (1994) economy. Household preferences are the same as Section 3, but the per-period budget constraint is now

$$c_{ht} + x_{ht} = z_{ht} + R_t k_{ht},$$

where x_{ht} is investment by household h and R_t is the rental price of capital. Each household faces a borrowing constraint, so $k_{ht} \geq 0$. The labor income process is the same as in Section 3. Each household's capital stock follows

$$k_{ht+1} = (1 - \delta)k_{ht} + x_{ht}.$$

There is no public debt or taxes.

The aggregate resource constraints are as in (15)-(17). Aggregate output is produced by a perfectly competitive representative firm that hires labor and capital on competitive spot markets. The rental price of capital clears the capital market: $\int_0^1 k_{ht} dh = k_t$.

We calibrate capital's share of GDP to be $\alpha = 0.35$, the depreciation rate δ to match a capital-to-output ratio (quarterly) of 14, and the discount factor β to match a steady-state annual interest rate of $r = 0.05$. We set the standard deviation of idiosyncratic income risk, σ, to match the same cross-sectional variance of consumption as in the baseline model without capital.

We calculate misallocation, A, using Proposition 5, assuming that the economy starts in the steady-state of the model with incomplete markets. The results are plotted in Figure 5 as a function of idiosyncratic risk. Misallocation at the benchmark values is 19 log points, similar to the calibration of the baseline model without capital.

As is well-known, in equilibrium, households overinvest in capital relative to first-best allocations due to a precautionary motive. To quantify the importance of overinvestment for misallocation, we compare misallocation in the benchmark economy to one where we impose that the capital stock remains constant. That is, we use the consumption possibility set that keeps the aggregate capital stock constant and equal to its status quo value

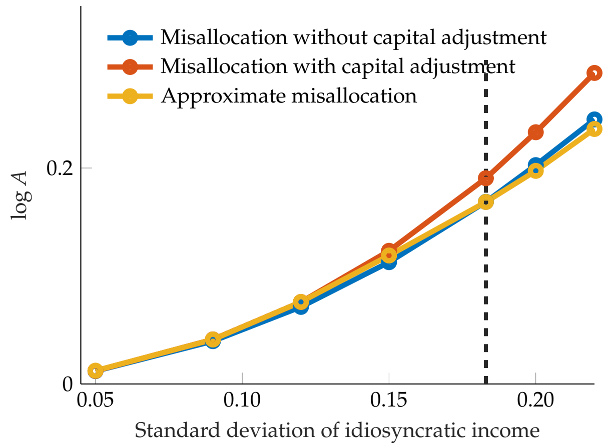


Figure 5: Misallocation as a function of idiosyncratic income risk. Dashed line is benchmark calibration.

in Definition 1. This means we can use equation (5) rather than (18). As expected, the distance to the frontier holding the capital stock constant is smaller than the distance to the frontier allowing the capital stock to adjust — and the gap grows as idiosyncratic risk, and the strength of the precautionary motive, rise. At the benchmark values, allowing for the aggregate capital stock to adjust raises the distance from the efficient frontier from 17 log points (holding capital fixed) to 19 log points. So in this calibration, roughly 10 percent of misallocation (2 of 19 log points) comes from inefficient capital accumulation, with the remainder from imperfect risk sharing.

For completeness, Figure 5 also reports the second-order approximation in Proposition 4, which continues to perform very well as an approximation to the case where the aggregate capital stock is held constant.[20]

5 Incompleteness of International Markets

In this section, we quantify misallocation in an environment with international trade in goods and assets. Relative to the baseline model in Section 3, we now allow each household to consume a different basket of goods, capturing home bias arising from preference heterogeneity and nontraded goods (e.g., households in Germany consume a different bundle than households in China).

[20]The gap between the blue and red lines in Figure 5 is relatively straightforward to characterize, but we omit it for brevity. Intuitively, it is the sum of Harberger triangles associated with the deviations of aggregate consumption from first-best due to the presence of an aggregate wedge in the aggregate Euler equation. The size of this aggregate wedge is the gap between the interest rate in the initial equilibrium and $1/\beta - 1$.

5.1 Environment with Heterogeneous Consumption Baskets

Recall that in Section 3, we did not need to fully specify the within-period production structure of the economy. This was because primary factors were inelastically supplied and all households had identical homothetic preferences. This meant that risk-sharing wedges had no effect on production decisions within each period. However, since now we allow households to consume different consumption baskets, risk-sharing wedges between households do change the combination of goods that are produced in each period and state. Therefore, we now spell out the production structure in more detail.

Preferences. Household h has intertemporal preferences:

$$u_h(\boldsymbol{c}_h) = \frac{1}{1-1/\eta} \sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}(s)^{1-\frac{1}{\eta}}.$$

Here, $c_{ht}(s)$ denotes consumption of household h at time t in state s. The crucial difference, relative to the baseline model in Section 3, is that each household h can consume a potentially different composite consumption good consisting of traded and nontraded varieties.

Technologies. In every period, t, of every state s, there is a set F of primary factor endowments and N of goods. Later, we will map F to 'country-specific labor and capital' and N to 'country-industry pairs.' The factors are inelastically supplied and owned by households, and used by producers in the same period (i.e. labor from t cannot be used by producers in $t+1$). Producer $i \in N$ has a CES production function that uses intermediate inputs and primary factor endowments with elasticity of substitution θ_i. Hence, the production function of i is

$$y_{it}(s) = z_{it}(s) \left(\sum_{j \in N} \alpha_{ij} \left(y_{ijt}(s)\right)^{\frac{\theta_i - 1}{\theta_i}} + \sum_{f \in F} \alpha_{if} \left(l_{ift}(s)\right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}},$$

where $z_i t(s)$ is a Hicks-neutral productivity shifter, and $y_{ijt}(s)$ and $l_{ift}(s)$ are intermediate input j and factor input f. The scalars α_{ij} and α_{if} are share parameters that affect expenditure shares across inputs for each i.

Note that this structure is general enough to accommodate any pattern of nested-CES producers. This model also accommodates any Armington-style model of trade, where productivity shifters, $z_{it}(s)$, for specialized intermediaries of imports and exports represent iceberg costs of trade. Without loss of generality, we treat the consumption bundle of each household $c_{ht}(s)$ as if it is produced by one of the goods producers and order the consumption goods first among the commodities in N.[21]

Resource constraints. The resource constraints of the economy are as follows: consumption of good h equals its production,

$$y_{ht}(s) = c_{ht}(s), \qquad (h \in H)$$

use of intermediate input i equals its production,

$$\sum_{j \in N} y_{jit}(s) = y_{it}(s), \qquad (i \in N, i \notin H)$$

use of factor f equals its endowment

$$\sum_{j \in N} l_{jft}(s) = z_{ft}(s), \qquad (f \in F).$$

Given these technologies and resource constraints, denote the dynamic consumption possibility set of the economy by $\mathcal{C}(1)$. Each element of $\mathcal{C}(1)$ is a vector of state-contingent consumption streams for every household. The model in Section 3 is the special case where all households consume the same good.[22]

5.2 Approximate Characterization

Proposition 1 applies to this economy, but computing misallocation this way requires fully specifying technologies, including, for example, the productivity processes. Instead, we provide a (second-order) approximation for misallocation that does not require as much information to implement. We do this by using wedges to decentralize the distorted incomplete-markets allocation, as in Section 3.

[21] For more details, see the discussion of the "standard-form" representation of nested-CES economies in Baqaee and Farhi (2019).

[22] We also assume throughout that no country is in technological autarky. Or equivalently, if some country is in technological autarky, then it is excluded from the analysis. This is because if a country is in autarky (e.g., the iceberg costs are infinite or there are no gains from trade), then that country is unaffected by incompleteness of financial markets since there is no way to transfer resources to that agent or insure them against fluctuations. Hence, by definition $A = 1$ if we include a country in technological autarky in the analysis, in our international exercise.

Denote the *wedge*, which is an implicit tax, on the consumption of household h at time t in state s by $\mu_{ht}(s)$. The intertemporal budget constraint for household h, in the decentralization with wedges, is

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)p_{ht}(s)c_{ht}(s) \leq I_h,$$

where $q_t(s)$ is the price of an Arrow security, $p_{ht}(s)$ is the consumption price index for household h in state s at time t (excluding the wedge), and I_h is initial wealth (including factor endowments and revenues from consumption tax wedges).

We now extend the definition of general equilibrium with wedges. Since our focus is on misallocation from incomplete markets for households, we abstract from other possible distortions and assume that firms set prices equal to marginal cost.

Definition 4 (Equilibrium with Wedges)**.** A general equilibrium with wedges is the collection of prices and quantities such that: (1) the price of each good i equals its marginal cost of production; (2) each producer takes prices as given and chooses quantities to maximize profits; (3) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and income as given; (4) household h earns income from primary factors and lump-sum taxes or transfers; (5) all resource constraints are satisfied.

The following proposition extends Proposition 3 to a setting with heterogeneous preferences. It shows that any feasible consumption allocation that is the equilibrium of a model with incomplete financial markets (and no other distortions) can be decentralized using only household-state-date-specific consumption taxes.[23] The following generalizes Proposition 3 to allow for preference heterogeneity.

Proposition 6 (Decentralization with Wedges)**.** *Consider an initial equilibrium consumption allocation $\mathbf{c}^0$. Assume that for each period t and state s, the consumption vector $\{c^0_{ht}(s)\}_{h\in H}$ is statically efficient. Then, for any $\bar{h} \in H$, setting*

$$\log \mu_{ht}(s) = -\frac{1}{\eta}\left[\log \frac{\omega_{ht}(s)/\omega_{h0}}{\omega_{\bar{h}t}(s)/\omega_{\bar{h}0}}\right] + \frac{1-\eta}{\eta}\left[\log \frac{p_{ht}(s)/p_{h0}}{p_{\bar{h}t}(s)/p_{\bar{h}0}}\right] \tag{19}$$

[23] We assume that the real exchange rate reflects the relative marginal cost of consumption goods. Aguiar et al. (2025) argue that real exchange rates may not reflect relative marginal costs due to the presence of other frictions. We abstract from such considerations in this paper but our methodology can be extended to cover additional wedges, including wedges in production, given an identification strategy for these additional wedges. As previously mentioned, the metric that Aguiar et al. (2025) use to evaluate the efficiency of international risk sharing is different to ours.

implies that $\boldsymbol{c}^0$ is a general equilibrium with those wedges, where $\omega_{ht}(s)$ is the expenditure of consumer h in period t and state s as a share of total household spending in that period and state, and $p_{ht}(s)$ is the consumption price index for this household in the initial equilibrium.[24] *The equilibrium allocation is supported by some lump-sum transfers across households.*

By construction, for some household $\bar{h}$, the wedge $\mu_{\bar{h}t}(s)$ equals 1 for every t and s. However, this choice of $\bar{h}$ has no bearing on any of the results, since only relative wedges matter for equilibrium allocations (see proofs in the appendix). Moreover, we do not need to specify the lump-sum transfers explicitly. For our purposes, all that matters is that there exist an equilibrium with the wedges in (19), and appropriate transfers, that can support $\boldsymbol{c}^0$ as an equilibrium allocation.

We refer to $\log \mu_{ht}(s)$ as *Backus-Smith* wedges. This is because when $\log \mu_{ht}(s)$ are all equal to zero, then the Backus and Smith (1993) condition for efficient risk sharing holds: households whose consumption prices grow relatively more quickly (i.e. whose real exchange appreciates) experience relatively faster growth in consumption expenditures if $\eta < 1$ (and slower growth in consumption quantities for any η).

If $\eta = 1$, then efficient allocations, where Backus-Smith wedges are zero, feature constant consumption expenditure shares over time, even though households consume different goods. This is related to the observation by Cole and Obstfeld (1991) that an economy with $\eta = 1$ and constant expenditure shares in equilibrium delivers efficient risk sharing even under financial autarky.

Given these wedges, we can now generalize Proposition 4 to this environment. To do so, denote the deviations of productivity shifters from some constant values by $\Delta \log \boldsymbol{z}$. That is, for producer i at time t in state s,

$$\Delta \log z_{it}(s) = \log \frac{z_{it}(s)}{\bar{z}_i},$$

where $\bar{z}_i$ is some constant (over time and states) level of productivity for producer i.

The following proposition approximates misallocation losses in terms of Harberger deadweight loss triangles.

Proposition 7 (Approximate Misallocation for International Model)**.** *Misallocation is approximately*

$$\log A \approx \frac{1}{2} \sum_{h \in H} \omega_h \sum_{h' \in H} \mathcal{M}_{hh'} \operatorname{Cov}_{r,\pi}\left[\, \log \mu_{ht}(s), \log \mu_{h't}(s) \,|\, h, h'\right],$$

[24] More precisely, $\omega_{ht}(s) = (\sum_{i \in N} p_{it}(s) c^0_{iht}(s)) / (\sum_{h' \in H} \sum_{i' \in N} p_{i't}(s) c^0_{i'h't}(s))$ where i indexes different goods, using equilibrium producer prices (i.e. exclusive of household-specific wedges) $p_{it}(s)$, and consumption quantities $c^0_{iht}(s)$.

where $\log \mu_{ht}(s)$ *is given by Proposition 6,* $\text{Cov}_{r,\pi}\left[\log \mu_h, \log \mu_{h'}\right]$ *denotes the covariance between* $\log \mu_{ht}(s)$ *and* $\log \mu_{h't}(s)$ *using* $\frac{r}{(1+r)^{t+1}}\pi(s)$ *as weights,* ω_h *is the expenditure share of household* h *at any date or state,* r *is the risk-free rate at any date or state, and the scalar* $\mathcal{M}_{hh'}$ *depends only on the static input-output matrix and elasticities of substitution (including* η*). The approximation error is of order* $(\log \boldsymbol{\mu})^3$ *and* $(\log \boldsymbol{\mu})^2 \Delta \log \boldsymbol{z}$*.*[25]

The explicit formula for the $H \times H$ matrix $\mathcal{M}$ in terms of the input-output table and elasticities of substitution is given in the appendix. The key is that the matrix $\mathcal{M}$ does not depend on either the date or the state. The scalar $\mathcal{M}_{hh'}$ encodes how a wedge on household the consumption of household h' distorts the consumption of household h from the perspective of the objective function in Proposition 1. The closed-economy Proposition 4 is a special case of Proposition 1 where $\mathcal{M}$ is the identity matrix multiplied by $-\eta$.

Proposition 7 is a sufficient statistics formula: misallocation can be approximated conditional on knowledge of the (static) input-output table at some date, elasticities of substitution (include η), the risk-free rate r at one point in time, and wedges, $\log \mu_{ht}(s)$, which are recoverable from Proposition 6. Importantly, one does not need to know the process driving productivity shocks $\Delta \log \boldsymbol{z}$ or the income process (which also depend on asset portfolios and returns, etc.) for each country.

Proposition 7 requires that in each period and state, the consumption vector is statically efficient (no within-period production distortions). This helps isolate the role of imperfect consumption smoothing. A more general version of this proposition could incorporate additional wedges on the producer side, see Baqaee and Burstein (2025b) for how to do this.

To build some intuition for Proposition 7, consider the following simple example.

Example 2 (Symmetric country example). Consider two symmetric countries, $h \in \{1, 2\}$, and suppose that each country produces a country-specific good using a linear technology from the local factor endowment (i.e. there is one industry in each country and no intermediate inputs). Let α denote the import share in both countries in the first date in status quo and let θ denote the elasticity of substitution between domestic and foreign goods.

Then applying Proposition 7 and rearranging yields

$$\log A \approx \frac{1}{2}\left[\frac{\alpha(1-\alpha)}{4\alpha(1-\alpha)\left(\frac{1}{\eta}-\frac{1}{\theta}\right)+\frac{1}{\theta}}\right]\text{Var}_{r,\pi}[\log \mu_{1t}(s)],$$

[25]Formally, $\text{Cov}_{r,\pi}\left[\log \mu_h, \log \mu_{h'}\right] \equiv \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\left(\log \mu_{ht}(s)-\log \bar{\mu}_h\right)\left(\log \mu_{h't}(s)-\log \bar{\mu}_{h'}\right)\right]$, where $\log \bar{\mu}_h = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{r}{(1+r)^{t+1}}\log \mu_{ht}(s) \mid h\right]$.

where $\log \mu_{1t}(s)$ is the Backus-Smith wedge on country 1 relative to country 2. This expression illustrates several important lessons. First, holding the Backus-Smith wedges, $\log \boldsymbol{\mu}$, constant, misallocation goes to zero if either the import share of consumption, α, approaches zero or one. When α is near 0 or 1, each country's consumption is almost entirely in one variety, so there is very little scope to reallocate varieties across households for insurance; risk sharing is effectively shut down. Second, as the elasticity of substitution between domestic and foreign goods, θ, rises to infinity, misallocation rises because there is more scope for international risk sharing when foreign and domestic goods are more substitutable.

Comparative statics in η are more subtle. The direct effect, holding the Backus-Smith wedges constant, is that misallocation falls as EIS, η, tends to zero. This is because consumption choices do not respond to wedges when the EIS is close to zero and so the wedges do not cause misallocation. However, the Backus-Smith wedges in (19) are themselves functions of η (given data on consumption expenditures and real exchange rates). In particular, these wedges explode as η goes to zero. Intuitively, since consumption choices are insensitive to wedges when η is low, we require very large consumption wedges to justify deviations from perfect risk sharing. This second effect always dominates (because it is order $1/\eta^2$), so misallocation is larger for lower values of η holding fixed the data. The comparative statics that misallocation is higher when η is lower, when θ is higher, and when import shares are higher, are all repeated in the empirical application in Section 6. In this application, we treat each country as a different 'household' h, recover $\mu_{ht}(s)$ from data on nominal consumption and real exchange rates using (19), and then apply Proposition 7 using the world input-output matrix as the source of $\mathcal{M}$.

5.3 Numerical Example

To gauge the accuracy of Proposition 7, we provide a numerical example. We simulate an Armington model of trade with 15 heterogeneous countries and homogeneous consumers within each country. Each country produces a differentiated good, the elasticity of substitution between goods of different origin is 3, the annual risk-free rate is $r = 5\%$, and the EIS is 0.5. We randomize the country sizes and the input-output matrices. We draw productivity shocks and Backus-Smith wedges from a multivariate lognormal distribution.

We vary the standard deviation of the productivity and wedge processes in Figure 6 and plot the exact gains from completing financial markets, computed using Proposition 1, and the approximate gains, computed using Proposition 7. To estimate the time-zero expectations in Proposition 7, we simulate the model for 100 years and treat the observed realizations as one sample path from the distribution generating the data. Since we only have one realization of the sample path, we estimate the expectation using this single observation. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to the average of the observations in the last five years of the data.

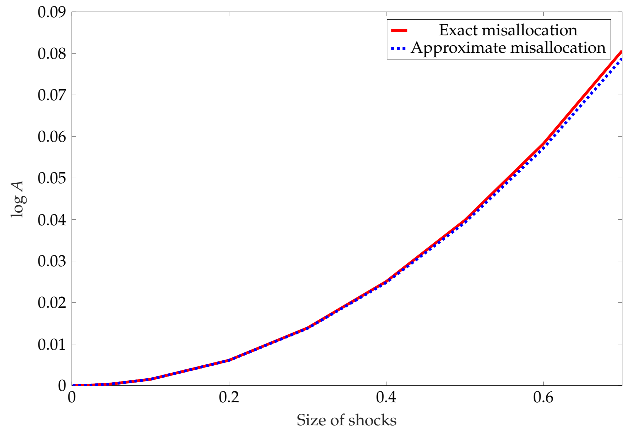


Figure 6: Performance of second-order approximation in Proposition 7. The x-axis is the standard deviation of the log productivity and Backus-Smith wedge shocks.

Even with very large shocks, the second-order approximation performs very well. Notably, to compute the second-order approximation, we do not need to know the stochastic process driving either the wedges or the productivity shocks.

6 Empirical Applications

We provide two empirical applications. The first one quantifies misallocation due to incomplete risk sharing in the United States using household consumption panel data from the Panel Study of Income Dynamics (PSID) and the results in Section 3. The second application quantifies misallocation due to imperfect consumption smoothing across countries using international macroeconomic data and the results in Section 5, abstracting from within-country household heterogeneity.

Since we use our sufficient statistics approximations, we can calculate misallocation using only the data on expenditures and prices as well as estimates of elasticities of substitution — we do not specify the details of financial market imperfections or the determinants of the consumption possibility set like the technology and income processes.

6.1 Misallocation from Domestic Financial Market Incompleteness

In this application, we quantify misallocation from the lack of complete domestic insurance markets in the US. Recall that our measure of misallocation is how much consumption (in every date and state) is left over if domestic insurance markets work perfectly and every household is kept indifferent relative to their status quo allocation in the data. The larger is this number, the greater is the extent of misallocation due to incomplete risk sharing. To avoid confusion, keep in mind that when we refer to the allocation in the data, we mean the entire state- and date-contingent equilibrium consumption process for each household starting in the first period, not just the consumption in that first period.

Approach. We use Proposition 4 to examine the extent of misallocation. We assume the data arise from an economy that meets the assumptions laid out in Section 3 — every household has the same static consumption aggregator and production is efficient in a static sense, but consumption allocations may be Pareto inefficient over time or across different states of nature. In particular, this means that we abstract from labor-leisure choice at the individual level, and capital accumulation at the aggregate level. Nevertheless, our quantitative results in Section 4 give some assurance that abstracting from these two margins is quantitatively innocuous for our measure $\log A$ and that our approximation formula continues to perform well even if these ingredients are added to the model.

Description of data. We use the PSID, which is a longitudinal panel survey of American households. We use a balanced panel of households from 1999 to 2021 with $2,096$ households. We use household consumption expenditures across six consumption categories collected once every two years. These categories are food (at home and away), child care, healthcare, education, transportation, and housing. We leave out other expenditure categories (like clothing and electronics) that are not collected in every wave. Housing expenditures do not measure owner-occupied rental value for homeowners, so we use the methodology in Baqaee et al. (2024) to impute owner-occupied housing costs.[26]

Mapping data to terms in Proposition 4. To apply Proposition 4, we set the EIS $\eta = 0.5$ (which is in line with estimates from the microeconomic literature) and the risk-free rate $r = 0.05$. We set ω_h to be household h's share of total consumption expenditures in 1999.[27]

[26] Briefly, we regress rent on observables for non-home-owners, and then use the estimates to predict equivalent owner-occupied rents for home-owners.

[27] We experimented with using contemporaneous shares ω_{ht} every period instead of freezing them in 1999 and the results are very similar.

We perform sensitivity analysis with respect to these choices when we present our results.

To estimate the variance of household log consumption shares over time and states, $\text{Var}_{r,\pi}[\log \omega_{ht}(s) \mid h]$, we replace the full conditional expectation with a linear predictor of consumption shares by period (i.e. the best possible nonlinear predictor). Specifically, for each horizon t, we regress household h's log consumption share on its date-zero consumption share (the first year of the sample, 1999) and a vector of household characteristics:[28]

$$\log \omega_{ht} = \gamma_t \log(\omega_{h0}) + \boldsymbol{\psi_t X_{h0}} + \epsilon_{ht}. \tag{20}$$

We then use the fitted values from (20) to construct household h's variance of log consumption shares, with each time period weighted by the discount factor implied by r. Because the sample does not span the full infinite horizon, we impute unobserved future terms using the last observed fitted value.[29] As shown in Figure 2, this imputation performs well in small samples.

Results. Figure 7 plots estimated misallocation losses in the PSID as a function of the annual interest rate. For our benchmark interest rate of 5%, misallocation is $\log = 0.23$. That is, the gains from eliminating idiosyncratic consumption volatility are $1 - \exp(-0.23) \approx$ 20 percent of consumption in every period and state. We can contrast this with the gains from eliminating aggregate volatility for a representative agent, which Lucas (1987) famously estimated to be three orders of magnitude smaller (0.05%).

The estimated gains in the PSID are very similar to those from the off-the-shelf calibration of the Bewley (1972) model in Section 3.4, even though the calibration does not target the moments of the consumption process. As expected, estimated misallocation decreases as the risk-free rate, or degree of impatience, rises. This is because deadweight loss triangles in the future are more heavily discounted. In the limit, as $r \to \infty$, households are infinitely impatient, there is no possibility to share risk, and misallocation is zero.

[28] The covariates are household wealth, state of residence, household size, home ownership status, household head's age, race, ethnicity, and college degree status, business assets of the head and spouse, household head's labor income, and spouse's labor income.

[29] Specifically, let $\hat{\omega}_{ht}$ denote the fitted value from (20). We estimate household h's discounted variance as $\text{Var}_{r,\pi}[\log \omega_{ht}(s) \mid h] = r \sum_{t=0}^{\infty} (1+r)^{-(t+1)} \left(\log \hat{\omega}_{ht} - \log \bar{\omega}_h\right)^2$, where $\log \bar{\omega}_h = r \sum_{t=0}^{\infty} (1+r)^{-(t+1)} \log \hat{\omega}_{ht}$, and we set $\log \hat{\omega}_{ht} = \log \hat{\omega}_{h,2021}$ for all $t > 2021$. The results are very similar if, instead, we set unobserved terms equal to the average fitted value over the last three years of the sample.

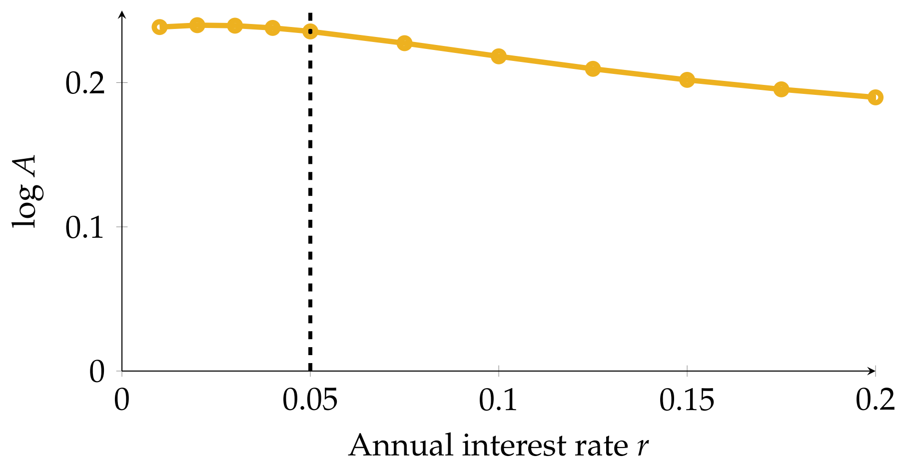


Figure 7: Estimated misallocation in PSID. Dashed line is the benchmark.

6.2 Misallocation from International Financial Market Incompleteness

The next application quantifies misallocation from the lack of complete international financial markets. More precisely, we calculate how much of every consumption good (in the world) is left over if financial markets are completed and agents in every country are kept indifferent relative to the status quo consumption process. The larger is this number, the greater is the extent of misallocation from incomplete risk sharing.

While Proposition 7 allows for imperfect consumption smoothing within and across countries, in this quantitative exercise, we abstract from within-country heterogeneity and treat each country as a single household. Hence, the misallocation we measure here arises solely from incomplete international risk sharing, not from incomplete risk sharing within countries. Computing an all-inclusive measure that allows for imperfect consumption smoothing across and within borders is conceptually straightforward, but would require having consumption panel data for every country.

Approach. We use Proposition 7, which allows for heterogeneity in household consumption bundles. We assume the data is generated by an economy satisfying the assumptions in Section 5 — allocations are efficient from a static perspective, but potentially inefficient over time and states of nature. The advantage of using the second-order approximation in Proposition 7, versus writing a fully-specified structural model and applying the exact result in Proposition 1, is that the informational requirements are much weaker. Specifically, we can apply Proposition 7 without taking a stance on the stochastic process driving either the wedges or the productivity shifters. For example, there may be productivity changes that we did not explicitly model, like changes in iceberg costs at the industry-country pair level, and they would not alter the validity of the second approximation.

Description of data. We use the 2014 release of the world input-output database (Timmer et al., 2015), and country-level nominal consumption data and CPI-based real exchange rates from the Global Macro Database from Müller et al. (2025) from 1970 to 2019.

Mapping data to terms in Proposition 7. We specialize the technologies introduced in Section 5 as follows. There are 32 countries (households), 54 industries in each country, and one primary factor endowment per country (i.e. labor equipped by capital).[30] The static preferences of household h are an h-specific Cobb-Douglas aggregator across different industries. Consumption by h from industry i is an (h, i)-specific Armington CES aggregator over different origin countries, with elasticity of substitution θ_T.

The production function of industry i in country h is a Cobb-Douglas aggregator of the local primary factor and an (h, i)-specific bundle of intermediate inputs from other industries. This intermediate bundle is also an (h, i)-specific Cobb-Douglas aggregator. The industry-j input used by producers in country h is an (h, j)-specific Armington aggregator with elasticity θ_T across different origin countries.[31]

We calibrate the consumption share of each country ω_h using that country's share of total consumption, investment, and government expenditures. We calibrate the input-output matrix required for $\mathcal{M}_{hh'}$ using the transaction flows in the World Input-Output Database 2014. We measure Backus-Smith wedges using the formula in Proposition 6 together with nominal consumption data and CPI-based real exchange rates from the Global Macro Database.

To estimate the time-zero expectations in Proposition 7, we treat the wedges from 1970 to 2019 as one sample path from the distribution generating the data. Since we only have one realization of the sample path, we estimate the expectation using this single observation. Because we do not observe all terms in the infinite sums for present value calculations, we treat unobserved terms as equal to an average of the observations in the

[30] We drop the activities of private households as employers industry and the activities of extraterritorial organizations and bodies industry from the sample. The list of countries is Australia, Austria, Belgium, Brazil, Canada, Switzerland, China, Cyprus, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Hungary, Indonesia, India, Ireland, Italy, Japan, South Korea, Luxembourg, Mexico, Malta, Netherlands, Norway, Poland, Portugal, Sweden, Turkey, and the US.

[31] This means we assume the same country-composition of the intermediate input bundle by industry. For example, mining & quarrying and the manufacture of basic metals in Australia, have the same expenditure shares on rubber and plastic products from China relative to India. On other hand, the expenditure share of the rubber and plastic industry summed across all origins by the mining & quarrying industry in Australia can differ from that by the manufacture of basic metals industry.

last five years of the data (2015 to 2019).[32] This mirrors the procedure we used in our Monte Carlo simulation in Section 5.3. The log wedges are nonzero if changes in log relative consumption and real exchange rates between countries do not comove perfectly. The median correlation across countries between annual changes in the US real exchange rate and relative real consumption is 0.17, whereas perfect risk sharing implies that this correlation should be -1.

To compute the terms in Proposition 7, we assume an annual risk-free rate $r = 0.05$, an EIS $\eta = 0.5$, and a trade elasticity $\theta_T = 2$. We vary these parameters in our sensitivity analyses.

Results. Misallocation in our baseline calibration is $\log A = 0.052$ percent — that is, with complete insurance markets, every country can be made indifferent to the status quo allocation with roughly 5.1 percent of every good left over. Recall that the status quo allocation is not simply consumption in the first period of the data, but the entire date- and state-contingent consumption processes.

If we set value-added shares of sales to one in every country, so that trade in intermediates is shut down, then misallocation from incomplete international markets is 2.7 percent of world consumption, compared to 5.1 percent in our baseline calibration with full input-output linkages. Hence, roughly half of the baseline consumption-smoothing misallocation is mediated by intermediate inputs.

The extent of misallocation depends importantly on the Backus-Smith wedges of fast-growing countries. For example, if we set China's Backus-Smith wedges to zero rather than using the observed realizations in the data, misallocation falls to 1.6 percent. If we further set the wedges to zero for other fast growing Asian countries like South Korea, India and Indonesia, misallocation falls to 0.9 percent.[33] Beyond that, the results are fairly stable to setting the wedges of additional countries to zero. This suggests that the measured extent of misallocation is driven largely by the consumption wedges of large countries with unusually rapid growth. Intuitively, these countries would have wanted to borrow when they were relatively poor from countries with lower growth rates.

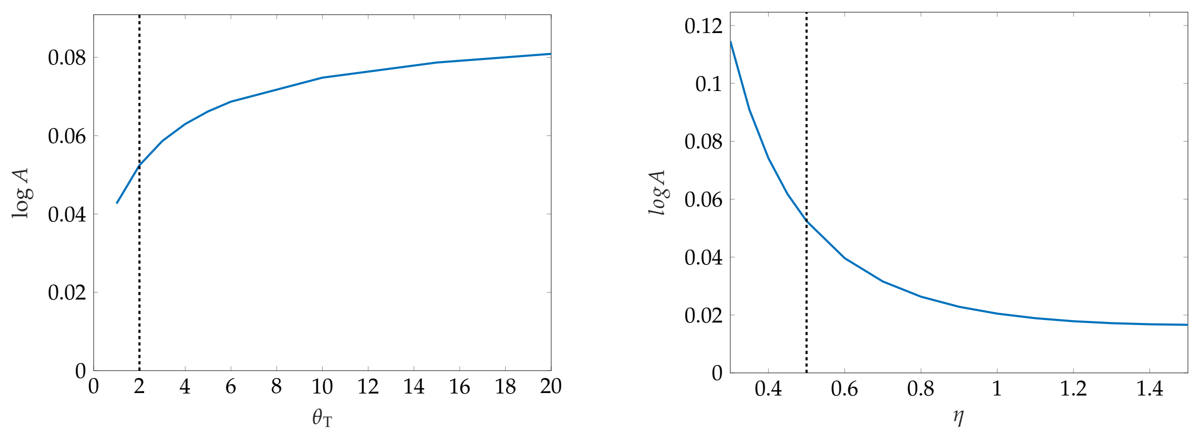
In the rest of this section, we report results including all 32 countries. We experimented with varying the start date, for example, if we start in 1980 instead of 1970, then misallocation is slightly larger at 6.3 percent. If we start in 1993, then we can increase the number of countries in the sample by including 10 additional countries that belonged to

[32]Results are very similar if we set unobserved terms equal to the last observed year of these terms.

[33]The results are very similar if, instead of setting the Backus-Smith wedges for these countries to zero, we drop them from the sample.

the Eastern Bloc. This raises misallocation to 6.6 percent.

We also vary the world input-output database year we use to calibrate the input-output table. If we use an earlier date, say 2006 instead of 2014, then misallocation is smaller, around 3.6 percent instead. This is because the world economy is less open in 2006 compared to 2014, so there is less scope for international risk sharing, as discussed in Example 2.



(a) Armington elasticity

(b) EIS

Figure 8: Misallocation as we vary parameters. Dashed line is benchmark value.

Figure 8 shows how our estimates of misallocation change as a function of the Armington trade elasticity and the EIS. As expected from Example 2, misallocation is larger the higher is the Armington elasticity, since more substitutability between domestic and foreign varieties facilitate more risk sharing; and misallocation is larger the lower is the EIS, since observed fluctuations in consumption are most costly for lower values of the EIS. Our estimates for misallocation are fairly insensitive to the Armington elasticity for the range the literature considers empirically plausible (e.g., from 1 to 5). However, our estimates are sensitive to lower values of the EIS. For example, if the EIS is 0.25, misallocation is around 10 percent — and these losses go to infinity as η approaches zero. We do not present graphs for how estimated misallocation varies as a function of the elasticity of substitution between industries or between intermediates and value added. Estimated losses are slightly increasing in these other elasticities.

7 Conclusion

We quantify misallocation arising from households' inability to perfectly share risks across states of nature and over time using a Debreu-style "distance to the frontier" measure.

Our efficiency measure is a multi-agent extension of Lucas's consumption-equivalent measure, is invariant to monotone transformations of utility, and does not rely on any particular social welfare function or on interpersonal utility comparisons. Using exact certainty-equivalent formulas and Harberger-triangle sufficient statistics, we show how to compute misallocation in a range of incomplete-market environments.

Our quantitative results imply that the welfare costs of market incompleteness are large, especially within countries. In our Bewley-type calibrations and in U.S. household consumption panel data (PSID), incomplete insurance against idiosyncratic risk is equivalent to destroying on the order of 20 percent of aggregate consumption in every date and state: if domestic financial markets were complete and households were fully compensated, roughly one-fifth of aggregate resources would be left over. These losses are about three orders of magnitude larger than Lucas's (1987) representative-agent estimate of the cost of aggregate consumption volatility, underscoring that most of the welfare cost of risk comes from uninsured idiosyncratic shocks rather than aggregate fluctuations.

By contrast, the costs of incomplete international risk sharing — treating each country as a representative agent — are modest. In our multi-country, multi-industry input-output application, incomplete international financial markets destroy about 5 percent of worldwide consumption, conditional on the observed path of consumption and prices. Moreover, most of this loss is driven by fast-growing economies such as China and India: excluding these countries reduces misallocation to roughly 1 percent, so risk-sharing gains among countries at similar levels of development are more limited. These findings highlight that the main global gains from more complete markets come from insuring household-level risks domestically and from intertemporal trade between fast-growing and slower-growing economies, rather than from additional risk sharing among similar advanced economies.

Methodologically, our analysis shows that the same wedge-and-triangle machinery that has become standard for studying production misallocation can be repurposed to study dynamic, stochastic misallocation of consumption. In our applications, misallocation can be approximated using only data on consumption expenditures and prices, a static input-output matrix, a risk-free rate, and a small set of substitution elasticities, without specifying income or productivity processes, asset portfolios, or detailed financial frictions. The framework we use in this paper is portable to other environments — such as economies with firm-side distortions or spatial models — and connects directly to the general productivity and misallocation framework in Baqaee and Burstein (2025a,b).

A promising area for future research is to extend our characterizations to study misallocation relative to the constrained-efficient Pareto frontier, accounting for the presence of other distortions and policy imperfections. This can be done by following the methodology outlined in Baqaee and Burstein (2025b).

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Appendix A Proofs

Proof of Proposition 1. By homogeneity of $\mathcal{C}(1)$, for any $Z > 0$,

$$\mathcal{C}\left(\frac{1}{Z}\right) = \frac{1}{Z}\,\mathcal{C}(1),$$

so any $\boldsymbol{c} \in \mathcal{C}(1/Z)$ can be written as $\boldsymbol{c} = \frac{1}{Z}\hat{\boldsymbol{c}}$ for some $\hat{\boldsymbol{c}} \in \mathcal{C}(1)$. Hence, (1) is equivalent to

$$A = \max\left\{a > 0 : \text{there exists } \hat{\boldsymbol{c}} \in \mathcal{C}(1) \text{ with } u_h\left(\frac{\hat{\boldsymbol{c}}_h}{a}\right) \geq u_h(\boldsymbol{c}_h^0) \text{ for every } h\right\}.$$

By Definition 2,

$$u_h\left(\frac{\hat{\boldsymbol{c}}_h}{a}\right) \geq u_h(\boldsymbol{c}_h^0) \quad \Longleftrightarrow \quad \tilde{u}_h(\hat{\boldsymbol{c}}_h) \geq a.$$

Therefore,

$$A = \max\left\{a > 0 : \text{there exists } \hat{\boldsymbol{c}} \in \mathcal{C}(1) \text{ with } \tilde{u}_h(\hat{\boldsymbol{c}}_h) \geq a \text{ for every } h\right\}.$$

□

Proof of Proposition 2. See text. □

Proof of Proposition 3. A decentralized equilibrium with wedges satisfies

$$\max_{c_{ht}(s)} \frac{1}{1-\frac{1}{\eta}} \sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}$$

subject to

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)c_{ht}(s) \leq I_h$$

and

$$\sum_h c_{ht}(s) = y_t(s).$$

Without loss of generality, we set the price of the consumption good to one in every period and state, $p_t(s) = 1$ (since only $p_t(s)q_t(s)$ is determined). Assume aggregate income is the numeraire. The first order conditions, resource constraints, budget constraint, and numeraire choice define equilibria:

$$\beta^t \pi(s) c_{ht}^{-\frac{1}{\eta}}(s) = \lambda_h q_t(s)\mu_{ht}(s).$$

$$\sum_h c_{ht}(s) = y_t(s).$$

$$\sum_s \sum_t q_t(s)\mu_{ht}(s)c_{ht}(s) = I_h$$

$$\sum_h I_h = 1.$$

We need to show that if

$$\mu_{ht}(s) = \left[\frac{\omega_{ht}(s)}{\omega_{h0}}\right]^{-\frac{1}{\eta}},$$

where $\omega_{ht}(s)$ are expenditure shares in the equilibrium with incomplete markets (the status quo) and ω_{h0} is the time-0 share in the equilibrium, then the following allocation is an equilibrium:

$$c_{ht}(s) = c^0_{ht}(s) = \omega_{ht}(s)y_t(s).$$

This requires showing that there is a set of ϕ_h, $q_t(s)$, and I_h such that all the equilibrium conditions are satisfied. Substituting in the wedges and the allocation into the first order condition yields the following restriction on ϕ_h and $q_t(s)$:

$$\phi_h q_t(s) = \beta^t \pi(s) y_t(s)^{-\frac{1}{\eta}} \left[\omega_{h0}\right]^{-\frac{1}{\eta}}.$$

Hence, dividing this equation for h' by H for some fixed household H gives

$$\phi_{h'} = \phi_H \left(\frac{\omega_{H0}}{\omega_{h'0}}\right)^{\frac{1}{\eta}}.$$

The resource constraint is satisfied automatically, since:

$$\sum_h c_{ht}(s) = \sum_h \omega_{ht}(s)y_t(s) = y_t(s).$$

Finally, substituting the FOC into the budget constraint yields

$$\frac{\sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s)}{\phi_h} = I_h.$$

Finally, the numeraire condition requires that

$$\sum_h I_h = 1.$$

Substituting the previous expression for I_h into this equation and rewriting every $\phi_{h'}$ in terms of ϕ_H for some h yields:

$$\sum_{h'} I_{h'} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{h'}} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_H \left(\frac{\omega_{H0}}{\omega_{h'0}}\right)^{\frac{1}{\eta}}} = 1.$$

Hence, we require that

$$\phi_H = \sum_h \left(\frac{\omega_{H0}}{\omega_{h0}}\right)^{-\frac{1}{\eta}} \sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s).$$

Since we can construct a collection of ϕ_h, $q_t(s)$, and I_h such that all equilibrium conditions are satisfied, with $c_{ht}(s) = \omega_{ht}(s) y_t(s)$ and $\mu_{ht}(s)$ given by (7), the proof is completed. □

Define a fictitious compensated agent as follows.

Definition 5. The *compensated agent* is an agent whose preferences are represented by

$$U(\boldsymbol{c}) = \min_h \{\tilde{u}_h(\boldsymbol{c}_h)\},$$

where $\tilde{u}_h(\boldsymbol{c}_h) = \left[u_h(\boldsymbol{c}_h)/u_h(\boldsymbol{c}_h^0)\right]^{\frac{\eta}{\eta-1}}$.

Define the compensated equilibrium as follows.

Definition 6 (Compensated Equilibrium)**.** A *compensated equilibrium* is the general equilibrium of an economy with the same technologies, resource constraints, and wedges as the original economy but where there is a representative agent with preferences as in Definition 5. For any equilibrium variable $X(t)$, denote the same variable in the compensated equilibrium by $X^{\text{comp}}(t)$.

Theorem 1 from Baqaee and Burstein (2025b) implies that aggregate efficiency can be calculated via the utility of the compensated agent in the compensated equilibrium.

Proof of Proposition 4. We begin by showing the following lemma, drawn from Baqaee and Burstein (2025b).

Lemma 1. *In the initial equilibrium (status quo), prices and quantities in the compensated equilibrium with wedges in Equation* (7) *coincide with those in the decentralized equilibrium with the same wedges.*

Proof of Lemma. A general proof can be found in Baqaee and Burstein (2025b). Here, we provide a self-contained derivation. We use this lemma both when households consume the same consumption good (as in the closed economy baseline) and when each household consumes a different bundle of goods (as in the international version of the model). Hence, in the proof of this lemma, we allow for consumption prices $p_{ht}(s)$ to vary by h, to allow for the possibility that households consume different consumption goods. The representative household maximizes

$$\min_h \{\tilde{u}_h(\boldsymbol{c}_h)\}$$

with

$$\tilde{u}_h(\boldsymbol{c}_h) = \left[\frac{\sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}}{\sum \beta^t \pi(s) c_{ht}^0(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}},$$

subject to a single budget constraint

$$\sum_h \sum_{s,t} q_t(s)\, p_{ht}(s) \mu_{ht}(s)\, c_{ht}(s) \leq I.$$

The solution to this problem can be found by two-step budgeting. The representative agent distributes income across h, and for each h maximizes $\tilde{u}_h(\boldsymbol{c}_h)$ subject to

$$\sum_{s,t} q_t(s)\, p_{ht}(s) \mu_{ht}(s)\, c_{ht}(s) = I_h.$$

The choice of $\{I_h\}_h$ in an interior equilibrium must be such that

$$\tilde{u}_h(\boldsymbol{c}_h) = \tilde{u}_{h'}(\boldsymbol{c}_h).$$

Setting $I_h = I_h^0$ for all h, where I_h^0 is the income level of household h in the status quo of the decentralized equilibrium with wedges gives the status quo allocation $\boldsymbol{c}^{\mathbf{0}}$, which satisfies $\tilde{u}_h(\boldsymbol{c}_h^0) = \tilde{u}_{h'}(\boldsymbol{c}_h^0) = 1$. □

Consider some exogenous parameter indexed by σ. For each value of σ, there is an endogenous set of wedges $\boldsymbol{\mu}(\sigma)$ that rationalize status quo allocations, and for some value of this parameter, normalized to be zero, the wedges are all equal to one: $\boldsymbol{\mu}(0) = \mathbf{1}$. Two examples are the standard deviation of idiosyncratic risk and the inverse EIS.

From Proposition 6 in Baqaee and Burstein (2025b), we know that, to a second-order approximation in σ, misallocation is given by

$$\begin{aligned}\log A &\approx -\frac{1}{2}\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\frac{d\log c_{ht}^{\text{comp}}(s)}{d\log \boldsymbol{\mu}} \cdot \frac{d\log \boldsymbol{\mu}}{d\sigma}\right] \frac{d\log \mu_{ht}(s)}{d\sigma}\Delta\sigma^2 \\ &= -\frac{1}{2}\sum_h \sum_{s,t} q_t(s) c_{ht}(s) d\log c_{ht}^{\text{comp}}(s) d\log \mu_{ht}(s), \end{aligned} \tag{21}$$

where we use the short-hand $d\log c_{ht}^{\text{comp}}(s)$ to mean $\frac{d\log c_{ht}^{\text{comp}}(s)}{d\log \boldsymbol{\mu}} \cdot \frac{d\log \boldsymbol{\mu}}{d\sigma}\Delta\sigma$ and $d\log \mu_{ht}(s)$ to mean $d\log \boldsymbol{\mu}/d\sigma\Delta\sigma$ in the compensated equilibrium evaluated at $\sigma = 0$. In this equation, we have used the convention that, in the compensated equilibrium with wedges, aggregate wealth is equal to one (this is our choice of numeraire).

Throughout, we use the fact that because of Lemma 1, we can use expenditures in the decentralized economy in place of expenditures in the compensated equilibrium with the same wedges that rationalize that allocation.

To calculate $d\log c_{ht}^{\text{comp}}(s)$, consider the first order condition of the representative agent in the compensated equilibrium:

$$\left[\frac{\sum_{t'} \beta^{t'} \pi(s) c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t'} \beta^{t'} \pi(s) c_{ht'}^{0}(s)^{1-\frac{1}{\eta}}}\right]^{\frac{\eta}{\eta-1}} \frac{\beta^t \pi(s) c_{ht}^{\text{comp}}(s)^{-\frac{1}{\eta}}}{\sum_{t'} \beta^{t'} \pi(s) c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} = \tilde{\phi}_h q_t^{\text{comp}}(s)\, \mu_{ht}(s)$$

where $\tilde{\phi}_h$ is the price index for $\tilde{u}_h$. Defining

$$\phi_h = \left[\tilde{\phi}_h \left(\sum_{t'} \beta^{t'} \pi(s) c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}\right)\right]^{-1}, \tag{22}$$

the first-order condition above can be written as

$$\tilde{u}_h\left(\boldsymbol{c}_h^{\text{comp}}\right) \beta^t \pi(s) c_{ht}^{\text{comp}}(s)^{-\frac{1}{\eta}} = \phi_h^{-1} q_t^{\text{comp}}(s)\, \mu_{ht}(s)$$

Taking ratios between h and H,

$$\frac{c_{ht}^{\text{comp}}(s)}{c_{Ht}^{\text{comp}}(s)} = \left(\frac{\mu_{ht}(s)\, \phi_h^{-1}}{\mu_{Ht}(s)\, \phi_H^{-1}}\right)^{-\eta} \left(\frac{\tilde{u}_h}{\tilde{u}_H}\right)^{\eta}. \tag{23}$$

Log differentiating and using the fact that the compensated representative agent keeps

$\tilde{u}_h / \tilde{u}_H$ constant, we have

$$d \log c_{ht}^{\text{comp}}(s) - d \log c_{Ht}^{\text{comp}}(s) = -\eta \left(d \log \frac{\mu_{ht}(s)}{\phi_h} - d \log \frac{\mu_{Ht}(s)}{\phi_H} \right), \tag{24}$$

The condition $d \log \tilde{u}_h = d \log \tilde{u}_H$ can be written as

$$\sum_{t,s} \frac{\beta^t \pi(s) c_{ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t',s'} \beta^{t'} \pi(s') c_{ht'}^{\text{comp}}(s')^{1-\frac{1}{\eta}}} d \log c_{ht}^{\text{comp}}(s) = \sum_{t,s} \frac{\beta^t \pi(s) c_{Ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t',s'} \beta^{t'} \pi(s') c_{Ht'}^{\text{comp}}(s')^{1-\frac{1}{\eta}}} d \log c_{Ht}^{\text{comp}}(s). \tag{25}$$

Throughout the rest of the proof, we use the fact that for any compensated equilibrium value X^{comp} we have

$$X_{ht}^{comp}(s)(\boldsymbol{z}, \boldsymbol{\mu}) \approx X_{ht}^{comp}(s)(\bar{\boldsymbol{z}}, \mathbf{1}) + \frac{dX_{ht}^{comp}}{d \log z} \Delta \log z + \frac{dX_{ht}^{comp}}{d \log \boldsymbol{\mu}} \cdot \Delta \log \boldsymbol{\mu}.$$

Hence, to evaluate any coefficients in the second-order approximation, we can use $X_{ht}^{comp}(s)(\bar{\boldsymbol{z}}, \mathbf{1})$ or $X_{ht}^{comp}(s)(\boldsymbol{z}, \boldsymbol{\mu})$ because the difference between them is first order, which when multiplied by the other terms in the Taylor expansion, will result in terms that are order three or higher.

Hence, we can evaluate (25) at $\sigma = 0$, and without aggregate productivity shocks ($z_t = \bar{z}$), where we know that $c_{ht}^{\text{comp}}(s) = c_h^{comp}$ for all h, giving us

$$\sum_{t,s} \beta^t \pi(s) d \log c_{ht}^{\text{comp}}(s) = \sum_{t,s} \beta^t \pi(s) d \log c_{Ht}^{\text{comp}}(s).$$

Substituting (24) into this, gives

$$\sum_{t,s} \beta^t \pi(s) \left(d \log \frac{\mu_{ht}(s)}{\phi_h} - d \log \frac{\mu_{Ht}(s)}{\phi_H} \right) = 0,$$

so

$$d \log \phi_h - d \log \phi_H = \frac{\sum_{t,s} \pi(s) \beta^t (d \log \mu_{ht}(s) - d \log \mu_{Ht}(s))}{\sum_{t''} \beta^{t'}}.$$

Plugging back into (24),

$$\begin{aligned} d \log c_{ht}^{\text{comp}}(s) - d \log c_{Ht}^{\text{comp}}(s) = & -\eta \left(d \log \mu_{ht}(s) - d \log \mu_{Ht}(s) \right) \\ & + \eta \frac{\sum_{t',s'} \pi(s') \beta^{t'} (d \log \mu_{ht'}(s') - d \log \mu_{Ht'}(s'))}{\sum_{t'} \beta^t}. \end{aligned} \tag{26}$$

Differentiating the resource constraint, at the status quo, with respect to σ gives

$$\sum_{h'} c^0_{h't}(s)\, d\log c^{\text{comp}}_{ht}(s) = \sum_{h'} c^0_{h't}(s) \left[\frac{d\log c^{\text{comp}}_{ht}(s)}{d\log \boldsymbol{\mu}} \cdot d\log \boldsymbol{\mu} \right] = 0,$$

where the second equation is a definition. Substituting (26) into the expression above and rearranging yields:

$$d\log c^{\text{comp}}_{ht}(s) = -\eta \left(d\log \mu_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \mu_{ht}(s)}{\sum_{t',s'} \beta^{t'} \pi(s')} - \sum_{h'} \omega_{h't}(s) \left(d\log \mu_{h't}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \mu_{h't}(s)}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right).$$

We also have

$$d\log \mu_{ht}(s) \equiv \frac{d}{d\sigma} \log \mu_{ht}(s) \Delta\sigma = -\frac{1}{\eta} \left[\frac{d\log \omega_{ht}(s)}{d\sigma} - \frac{d\log \omega_{h0}}{d\sigma} \right] \Delta\sigma \equiv -\frac{1}{\eta} \left[d\log \omega_{ht}(s) - d\log \omega_{h0} \right],$$

where the first and last equality are a notational convention. We can now substitute these back into our Harberger triangle formula, (21):

$$\begin{aligned}
\Delta \log A &\approx \frac{1}{2} \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\eta \left(d\log \mu_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \mu_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right.\right. \\
&\quad \left.\left. - \sum_{h'} \omega_{h't}(s) \left(d\log \mu_{h't}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \mu_{h't'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right) \right] d\log \mu_{ht}(s) \\
&= -\frac{1}{2} \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\left(d\log \omega_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \omega_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right.\right. \\
&\quad \left.\left. - \sum_{h'} \omega_{h't}(s) \left(d\log \omega_{h't}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \omega_{h't'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right) \right] d\log \mu_{ht}(s) \\
&= \frac{1}{2} \frac{1}{\eta} \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[d\log \omega_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \omega_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right] \\
&\qquad \times \left[d\log \omega_{ht}(s) - d\log \omega_{h0} \right] \\
&\quad - \frac{1}{2} \frac{1}{\eta} \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \omega_{h't}(s) \left(d\log \omega_{h't}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s')\, d\log \omega_{h't'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')} \right) \right] \\
&\qquad \times \left[d\log \omega_{ht}(s) - d\log \omega_{h0} \right].
\end{aligned}$$

$$= \frac{1}{2}\frac{1}{\eta}\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[d\log\omega_{ht}(s) - \frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)d\log\omega_{ht'}(s')}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right]$$

$$+\frac{1}{2}\frac{1}{\eta}\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega_{h't}\left(s\right)\frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)d\log\omega_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right].$$

where the final line uses the fact that $\sum_{h'}\omega_{h't}\left(s\right)d\log\omega_{h't}(s) \approx 0$. We now focus on the last line of the expression above, and show that it is equal to zero to a second-order approximation. To do so, re-write the last line in the expression above as $\frac{1}{2\eta}$ times

$$\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega_{h't}\left(s\right)x_{h'}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right], \tag{27}$$

where

$$x_{h'} \equiv \frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)d\log\omega_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}.$$

Define

$$\bar{\omega}_h \equiv \frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)\omega_{h't'}(s')}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}.$$

Note that

$$\omega_{ht}(s) \approx \bar{\omega}_h + \frac{d\omega_{ht}(s)}{d\sigma}d\sigma,$$

so

$$\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega_{h't}\left(s\right)x_{h'}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right]$$

$$= \sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\left[\bar{\omega}_{h'} + \frac{d\omega_{h't}(s)}{d\sigma}d\sigma\right]x_{h'}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right].$$

Dropping higher order terms gives

$$\sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\omega_{h't}\left(s\right)x_{h'}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right]$$

$$= \sum_h \sum_{s,t} q_t(s)c_{ht}(s)\left[\sum_{h'}\bar{\omega}_{h'}x_{h'}\right]\left[d\log\omega_{ht}(s) - d\log\omega_{h0}\right].$$

Next substitute $x_{h'}$ back in to get

$$\begin{aligned}
&\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} x_{h'}\right] [d\log \omega_{ht}(s) - d\log \omega_{h0}] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} \frac{\sum_{t',s'} \beta^{t'} \pi(s') [d\log \omega_{h't'}(s')]}{\sum_{t',s'} \beta^{t'} \pi(s')}\right] [d\log \omega_{ht}(s) - d\log \omega_{h0}] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta) \sum_{t',s'} \beta^{t'} \pi(s') \sum_{h'} \bar{\omega}_{h'} [d\log \omega_{h't'}(s')]\right] [d\log \omega_{ht}(s) - d\log \omega_{h0}] \\
&= \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta) \sum_{t',s'} \beta^{t'} \pi(s') \sum_{h'} \left[\omega_{h't}(s) - \frac{d\omega_{h't}(s)}{d\sigma} d\sigma\right] [d\log \omega_{h't'}(s')]\right] \\
&\times [d\log \omega_{ht}(s) - d\log \omega_{h0}].
\end{aligned}$$

Again drop higher order terms to get

$$\begin{aligned}
&\sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[\sum_{h'} \bar{\omega}_{h'} x_{h'}\right] [d\log \omega_{ht}(s) - d\log \omega_{h0}] \\
&\approx \sum_h \sum_{s,t} q_t(s) c_{ht}(s) \left[(1-\beta) \sum_{t',s'} \beta^{t'} \pi(s') \sum_{h'} \omega_{h't}(s) d\log \omega_{h't'}(s')\right] [d\log \omega_{ht}(s) - d\log \omega_{h0}] \\
&\approx 0,
\end{aligned}$$

since $\sum_{h'} \omega_{h't}(s) d\log \omega_{h't'}(s') \approx 0$. This allows us to write

$$\Delta \log A \approx \frac{1}{2\eta} \sum_{h,s,t} q_t(s) c_{ht}(s) \left(d\log \omega_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s') d\log \omega_{ht}(s)}{\sum_{t',s'} \beta^{t'} \pi(s')}\right) [d\log \omega_{ht'}(s') - d\log \omega_{h0}].$$

Next, we use the fact that, at $\sigma = 0$ and $z_t = \bar{z}$, we have $q_t(s) = \pi(s)(1+r)^{-t}/[\sum(1+r)^{-t'}\bar{y}]$ and $c_{ht}(s) = \omega_h \bar{y}$, where $1 + r = \beta^{-1}$. Note that the denominator in the Arrow security price is needed to ensure aggregate wealth is equal to one (our choice of numeraire). That is, $\sum_{s,t} q_t(s)\bar{y} = 1$. So, we can again, evaluate the coefficients at this point (since differences are higher order):

$$\begin{aligned}
&\Delta \log A \approx \\
&\frac{1}{2}\frac{1}{\eta} \sum_h \sum_{s,t} \frac{\pi(s)\beta^t}{\sum_{t'} \beta^{t'}} \omega_h \left(d\log \omega_{ht}(s) - \frac{\sum_{t',s'} \beta^{t'} \pi(s') d\log \omega_{ht'}(s')}{\sum_{t',s'} \beta^{t'} \pi(s')}\right) [d\log \omega_{ht}(s) - d\log \omega_{h0}]
\end{aligned}$$

Finally, we use the fact that

$$d\log\omega_{ht}(s) \approx \log\omega_{ht}(s) - \log\omega_h,$$

where $\log\omega_h$ is household $h's$ consumption share when $\sigma = 0$ (i.e. the point of approximation). Hence, if we substitute this into the Harberger formula and cancel, we get

$$\begin{aligned}
\Delta\log A &\approx \frac{1}{2}\frac{1}{\eta}\sum_h\sum_{s,t}\frac{\pi(s)\beta^t}{\sum_{t'}\beta^{t'}}\omega_h\left[\left(\left[\log\omega_{ht}(s)-\log\omega_h\right]-\frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)\left[\log\omega_{ht}(s)-\log\omega_h\right]}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}\right)\right] \\
&\quad\times\left[\left(\log\omega_{ht}(s)-\log\omega_h\right)-\left(\log\omega_h(s)-\log\omega_h\right)\right] \\
&= \frac{1}{2}\frac{1}{\eta}\sum_h\sum_{s,t}\frac{\pi(s)\beta^t}{\sum_{t'}\beta^{t'}}\omega_h\left[\left(\log\omega_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)\left[\log\omega_{ht}(s)\right]}{\sum_{t',s'}\beta^{t'}\pi\left(s'\right)}\right)\right] \\
&\quad\times\left[\log\omega_{ht}(s)-\log\omega_{h0}\right].
\end{aligned}$$

The term $\sum_{s,t}\frac{\pi(s)\beta^t}{\sum_{t'}\beta^{t'}}\omega_h\left[\left(\log\omega_{ht}(s)-\frac{\sum_{t',s'}\beta^{t'}\pi(s')[\log\omega_{ht}(s)]}{\sum_{t',s'}\beta^{t'}\pi(s')}\right)\right]\times\left[\log\omega_{ht}(s)-\log\omega_{h0}\right]$ is equal to the variance of $\log\omega_{ht}(s)$ for household h using $\frac{\pi(s)\beta^t}{\sum_{t'}\beta^{t'}}$ as weights. Equation (9) follows using the fact that $\omega_h = \omega_{h0}$ and $1 + r = \beta^{-1}$ at the point of approximation. □

Proof of Proposition 5. In this proof, we use $C_t(s)$ and $\boldsymbol{C}$ to denote the aggregate consumption process, instead of lower case $c_t(s)$ and $\boldsymbol{c}$, so as to avoid confusion with consumption allocations, $\boldsymbol{c} = \{\boldsymbol{c}_h\}$. Observe that consumption allocations on the Pareto frontier must satisfy $c_{ht}(s) = \lambda_h C_t(s)$ for some household-specific λ_h with $\sum_h \lambda_h = 1$. Define $\Gamma(k_0)$ to the set of feasible aggregate consumption paths given initial capital stock k_0. Then substituting this into (2), and manipulating yields

$$\begin{aligned}
A &= \max_{\boldsymbol{\lambda},\boldsymbol{C}}\min_h\left\{\tilde{u}_h(\boldsymbol{c}_h) : \{\boldsymbol{c}_h\}\in\mathcal{C}\right\}, \\
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}}\min_h\left\{\lambda_h\tilde{u}_h(\boldsymbol{C}) : \boldsymbol{C}\in\Gamma(k_0)\right\},
\end{aligned}$$

using the functional form for $\tilde{u}_h(\boldsymbol{c}_h) = \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t {c_{ht}}^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}}$, we have

$$
\begin{aligned}
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \lambda_h \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}, \\
&= \max_{\boldsymbol{\lambda},\boldsymbol{C}} \min_h \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}, \\
&= \max_{\boldsymbol{\lambda}} \min_h \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \right\} \max_{\boldsymbol{C}} \left\{ \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} : \boldsymbol{C} \in \Gamma(k_0) \right\}.
\end{aligned}
$$

Let $\boldsymbol{C}^*$ be the maximizing choice of aggregate consumption in the maximization problem above. Then, we know that $\lambda_h \tilde{u}_h(\boldsymbol{C}^*) = \lambda_{h'} \tilde{u}_{h'}(\boldsymbol{C}^*)$. Furthermore, we know that the solution to

$$
\max_{\boldsymbol{\lambda}} \min_h \left\{ \lambda_h \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}} \right)^{-\frac{\eta}{\eta-1}} \right\}
$$

implies that for some fixed h and every h'

$$
\lambda_h \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{h't}(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}} = \lambda_{h'},
$$

as well as

$$
\sum_{h'} \lambda_{h'} = 1.
$$

Combining these equations yields

$$
\lambda_h = \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{ht}(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c^0_{h't}(s)^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}},
$$

for every h. Hence,

$$
\begin{aligned}
A &= \lambda_h \tilde{u}_h(\boldsymbol{C}^*) \\
&= \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}} \left(\frac{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t^*(s)^{1-\frac{1}{\eta}}}{\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{ht}^0(s)^{1-\frac{1}{\eta}}}\right)^{\frac{\eta}{\eta-1}} \\
&= \frac{\left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t C_t^*(s)^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}{\sum_{h'} \left(\sum_s \pi(s) \sum_{t=0}^{\infty} \beta^t c_{h't}^0(s)^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}.
\end{aligned}
$$

Substituting this into (5) and using the definition of certainty equivalent yields the desired result. □

Proof of Proposition 6. Recall from Definition 4 that a general equilibrium with wedges is a collection of prices and quantities such that: (1) the price of each good i equals its marginal cost of production; (2) each producer takes prices as given and chooses quantities to maximize profits; (3) each household chooses consumption quantities to maximize utility taking prices, consumption tax wedges, and income as given; (4) household h earns income from primary factors and tax revenues; (5) all resource constraints are satisfied.

We show that given the wedges in (19), the status quo allocation, and status quo static relative prices constitute a decentralized equilibrium with wedges. We do this by showing that there is some distribution of household wealth and Arrow security prices (to be determined), such that the status quo allocations and prices maximize each household's problem, cause prices to equal marginal cost for each good, and statisfy all resource constraints.

By construction, the static relative prices satisfy the firms' first-order conditions and set prices equal to marginal costs. Furthermore, all resource constraints are satisfied by construction. Hence, we only need to check that the candidate equilibrium can satisfy the households' problem. Consider the households' problem:

$$
\max_{c_{ht}(s)} \frac{1}{1-\frac{1}{\eta}} \sum \beta^t \pi(s) c_{ht}(s)^{1-\frac{1}{\eta}}
$$

subject to

$$
\sum_s \sum_t q_t(s) p_{ht}(s) \mu_{ht}(s) c_{ht}(s) \leq I_h,
$$

where I_h is the household's wealth. We assume that aggregate wealth is the numeraire, so that

$$\sum_h I_h = 1.$$

Equation (19) implies that the wedges are given by:

$$\mu_{ht}(s) = \left[\frac{p^0_{ht}(s)\,/\,p^0_{h0}}{p^0_{\bar{h}t}(s)\,/\,p^0_{\bar{h}0}}\right]^{\frac{1-\eta}{\eta}} \left[\frac{\omega_{ht}(s)\,/\,\omega_{h0}}{\omega_{\bar{h}t}(s)\,/\,\omega_{\bar{h}0}}\right]^{-\frac{1}{\eta}}, \tag{28}$$

where $p^0_{ht}(s)$ denotes goods' prices in the status quo. In this equation, we explicitly distinguish between prices in the decentralized equilibrium with these wedges, $p_{ht}(s)$ and the prices in the primitive economy in status quo $p^0_{ht}(s)$. This abuse of notation is harmless, since, as we show in this proof, $p^0_{ht}(s)$ is consistent with equilibrium in the decentralized economy with wedges.

We show that the status quo allocation and static relative prices are decentralized equilibria by showing that all remaining equilibrium conditions, namely first-order conditions for utility maximization, household budget constraints, and the numeraire condition, can all be satisfied given these wedges, relative static prices, and quantities. To that end, in the decentralized equilibrium with wedges that we construct, the consumption allocation can be expressed as

$$c_{ht}(s) = c^0_{ht}(s) = \frac{\omega_{ht}(s)\,E^0_t(s)}{p^0_{ht}(s)},$$

where

$$E^0_t(s) = \sum_h p^0_{ht}(s)\,c^0_{ht}(s).$$

The first-order condition for $c_{ht}(s)$, given Lagrange multiplier ϕ_h and Arrow security prices $q_t(s)$ is

$$\beta^t \pi(s) c_{ht}^{-\frac{1}{\eta}}(s) = \phi_h q_t(s) p_{ht}(s) \mu_{ht}(s).$$

Substituting (28) into this first-order condition yields the following restriction on Lagrange multipliers ϕ_h and Arrow prices $q_t(s)$:

$$\beta^t \pi(s) p_{ht}^{\frac{1}{\eta}}(s) E_t^{-\frac{1}{\eta}}(s) \omega_{ht}^{-\frac{1}{\eta}}(s) = \phi_h q_t(s) p_{ht}(s) \left[\frac{p^0_{ht}(s)\,/\,p^0_{h0}}{p^0_{\bar{h}t}(s)\,/\,p^0_{\bar{h}0}}\right]^{\frac{1-\eta}{\eta}} \left[\frac{\omega_{ht}(s)\,/\,\omega_{h0}}{\omega_{\bar{h}t}(s)\,/\,\omega_{\bar{h}0}}\right]^{-\frac{1}{\eta}}. \tag{29}$$

We conjecture that $p^0_{ht}(s) = p_{ht}(s)$ is consistent with equilibrium. Then, dividing this equation for h' by $\bar{h}$ for some fixed household $\bar{h}$ allows us to express ϕ_h as a function of

$\phi_{\bar{h}}$:

$$\phi_{h'} = \phi_{\bar{h}} \left(\frac{\omega_{\bar{h}0}}{\omega_{h'0}} \right)^{\frac{1}{\eta}} \left(\frac{p^0_{h0}}{p^0_{\bar{h}0}} \right)^{\frac{1-\eta}{\eta}} . \tag{30}$$

Substituting the FOC into the budget constraint pins down household wealth as a function of Lagrange multipliers ϕ_h:

$$\frac{\sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s)}{\phi_h} = I_h. \tag{31}$$

The numeraire condition pins down $\phi_{\bar{h}}$:

$$\sum_{h'} I_{h'} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{h'}} = \sum_{h'} \frac{\sum_s \sum_t \beta^t \pi(s) c_{h't}^{1-\frac{1}{\eta}}(s)}{\phi_{\bar{h}} \left(\frac{\omega_{\bar{h}0}}{\omega_{h'0}} \right)^{\frac{1}{\eta}} \left(\frac{p^0_{h0}}{p^0_{\bar{h}0}} \right)^{\frac{1-\eta}{\eta}}} = 1$$

Hence, we require that

$$\phi_{\bar{h}} = \sum_h \left(\frac{\omega_{\bar{h}0}}{\omega_{h0}} \right)^{-\frac{1}{\eta}} \left(\frac{p^0_{h0}}{p^0_{\bar{h}0}} \right)^{\frac{\eta-1}{\eta}} \sum_s \sum_t \beta^t \pi(s) c_{ht}^{1-\frac{1}{\eta}}(s). \tag{32}$$

Hence, (32) pins down $\phi_{\bar{h}}$, (30) pins down ϕ_h for every other h, (31) pins down I_h, and (29), for any h, pins down the Arrow security prices $q_t(s)$ in the decentralized equilibrium with wedges that supports the status quo allocation. Since expenditures in the decentralized equilibrium with wedges coincides with expenditures in the status quo, and production technologies are all the same, and all firms set prices equal to marginal cost, static relative prices in the status quo are consistent with equilibrium, which confirms the conjecture above.

□

Proof of Proposition 7. We follow the same steps as in the proof of Proposition 4, but this time allowing for differences in consumption baskets across households. First, index allocations by an exogenous scalar parameter σ. We assume that allocations are a smooth function of σ, which means that $\boldsymbol{\mu}(\sigma)$, defined by (19) is also smooth. We assume that $\boldsymbol{\mu}(0) = \mathbf{1}$ and $\boldsymbol{z}_t(\sigma) = \bar{\boldsymbol{z}}$, so that at $\sigma = 0$, wedges are all equal to one and productivities are constant over time and state. For example, the parameter σ could index the standard deviation of h-level productivities.

As before, following Proposition 6 from Baqaee and Burstein (2025b), we express misallocation to a second-order approximation in the parameter σ as

$$\log A = -\frac{1}{2}\sum_{h}\sum_{s,t} q_t(s)p_{ht}(s)c_{ht}(s)d\log c_{ht}^{\text{comp}}(s)d\log \mu_{ht}(s). \tag{33}$$

Here, $d\log c_{ht}^{\text{comp}}(s)$ is a short-hand to mean $\frac{d\log c_{ht}^{\text{comp}}(s)}{d\log \boldsymbol{\mu}}\cdot\frac{d\log\boldsymbol{\mu}}{d\sigma}\Delta\sigma$ and $d\log\mu_{ht}(s)$ to mean $d\log\boldsymbol{\mu}/d\sigma\Delta\sigma$ in the compensated equilibrium evaluated at $\sigma = 0$, with $\boldsymbol{\mu}(0) = \mathbf{1}$. As before, we take aggregate wealth to be the numeraire.

The first-order conditions of the compensated agent imply

$$\frac{c_{ht}^{\text{comp}}(s)}{c_{\bar{h}t}^{\text{comp}}(s)} = \left(\frac{p_{ht}(s)\,\mu_{ht}(s)\,\phi_h^{-1}}{p_{\bar{h}t}(s)\,\mu_{\bar{h}t}(s)\,\phi_{\bar{h}}^{-1}}\right)^{-\eta}\left(\frac{\tilde{u}_h}{\tilde{u}_{\bar{h}}}\right)^{\eta}. \tag{34}$$

The derivation is very similar to that of equation (23) but allows for difference in households' consumption baskets.

We now solve for $d\log c_{ht}^{\text{comp}}(s)$ up to a first-order approximation. Log differentiate (34) and use the fact that the representative agent keeps $\tilde{u}_h/\tilde{u}_{\bar{h}}$ unchanged to obtain

$$d\log c_{ht}^{\text{comp}}(s) - d\log c_{\bar{h}t}^{\text{comp}}(s) = -\eta d\log\frac{p_{ht}^{\text{comp}}(s)}{p_{\bar{h}t}^{\text{comp}}(s)} - \eta\left(d\log\frac{\mu_{ht}(s)}{\phi_h} - d\log\frac{\mu_{\bar{h}t}(s)}{\phi_{\bar{h}}}\right), \tag{35}$$

where ϕ_h is household h's Lagrange multiplied defined in (22). This equation pins down final demand in the compensated economy as a function of relative prices in the compensated economy, wedges, and Lagrange multipliers.

To pin down these down, we must introduce some notation from the production side of the model. Define the within-period (static) $(H+N+F)\times(H+N+F)$ input-output matrix:

$$\Omega = \left[\begin{array}{ccc|ccc|ccc}
0 & \cdots & 0 & b_{11} & \cdots & b_{1N} & 0 & \cdots & 0 \\
\vdots & \cdots & \vdots & & \cdots & & & \cdots & \\
0 & \cdots & 0 & b_{H1} & \cdots & b_{HN} & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & \Omega_{11} & \cdots & \Omega_{1N} & \Omega_{1N+1} & \cdots & \Omega_{1N+F} \\
\vdots & \cdots & \vdots & & \ddots & & & & \\
0 & \cdots & 0 & \Omega_{N1} & & \Omega_{NN} & \Omega_{NN+1} & \cdots & \Omega_{NN+F} \\
\hline
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{array}\right]$$

The first H rows correspond to the households consumption baskets. The next N rows correspond to the expenditure shares of each producer on every other producer and factor. The last F rows correspond to the expenditure shares of the primary factors. Every row of Ω adds up to one or zero.

The Leontief inverse matrix is the $(H+N+F)\times(H+N+F)$ matrix defined as

$$\Psi \equiv (I-\Omega)^{-1}.$$

Let ω_h denote the share of expenditures of household h in total expenditures in a given period. That is,

$$\omega_{ht}(s) = \frac{p_{ht}(s)\, c_{ht}(s)}{\sum p_{h't}(s)\, c_{h't}(s)} = \frac{p_{ht}(s)\, c_{ht}(s)}{E_t(s)},$$

where $E_t(s)$ denotes total final expenditures in period t and date s (not including wedge revenues).

The within-period Domar weights are denoted by λ, where $\lambda_{it}(s)$ are the sales of i in period t and state s relative to total final expenditures, $E_t(s)$, in that date and state[34]

$$\lambda_{it}(s) = \frac{p_{it}(s) y_{it}(s)}{E_t(s)} \mathbf{1}[i\in N] + \frac{w_{it}(s) l_{it}(s)}{E_t(s)} \mathbf{1}[i\in F] + \frac{p_{it}(s) c_{it}(s)}{E_t(s)} \mathbf{1}[i\in H].$$

Market clearing identities imply that:

$$\lambda_t'(s) = \omega_t'(s) + \lambda_t'(s)\Omega_t(s) = \omega_t(s)'\Psi_t(s).$$

The following equations hold to a first-order in the compensated equilibrium. Goods' prices in a given period and state are given by the Leontief-inverse weighted changes in factor prices in that period and state:

$$d\log p_t^{\text{comp}}(s) = \sum_f \Psi_{(:,f)} d\log \lambda_{ft}^{\text{comp}}(s) + d\log E_t^{\text{comp}}(s), \tag{36}$$

where $\lambda_{ft}(s)$ is the sales of factor f in period t and state s relative to total final expenditures $E_t^{\text{comp}}(s) = \sum_h \mu_{ht}(s) p_{ht}^{\text{comp}}(s) c_{ht}^{\text{comp}}(s)$ and $\Psi_{(:,f)}$ is fth column of Ψ corresponding to factor f. We use the fact that changes in the wage of factor f in period t and state s is, in logs, the same as the change in expenditures on that factor (since factor quantity is held fixed in the variation). This equation is standard and follows from Shephard's

[34]Within-period Domar weights are sales in a period divided by total consumption expenditures in that period. We refer to these as within-period Domar weights to contrast them with Arrow-Debreu Domar weights, which are net present value sales divided by net present value of total consumption using Arrow securities.

lemma, factor market clearing in the compensated equilibrium, and the fact that $\sum_f \Psi_{(:,f)}$ is a vector of all ones.

Next, we note changes in the input-output matrix in period t and state s depend on changes in relative prices, which implies:

$$d\Omega_t^{\text{comp}}(s) = \Theta d\log\lambda_t^{\text{comp}}(s), \tag{37}$$

where Θ is some matrix involving cross-price elasticities and the Leontief inverse (we provide the specific formula for Θ below).

Finally, differentiating the identity that $\left(\lambda_t^{\text{comp}}(s)\right)' = (\omega_t^{\text{comp}}(s))'\Psi_t^{\text{comp}}(s)$ gives

$$\left(d\lambda_t^{\text{comp}}(s)\right)' = (\omega^{\text{comp}})'\Psi d\Omega_t^{\text{comp}}(s)\Psi + \left(d\omega_t^{\text{comp}}(s)\right)'\Psi \tag{38}$$

where we use the fact that at the point of approximation, $\Psi_t^{\text{comp}}(s) = \Psi_t(s)$ and where $\omega_{ht}^{\text{comp}}(s)$ denotes household h's share of total final spending in that period and state. Finally, by definition,

$$d\log c_t^{\text{comp}}(s) = d\log\omega_t^{\text{comp}}(s) - \left[d\log p_t^{\text{comp}}(s) - d\log E_t^{\text{comp}}(s)\right]. \tag{39}$$

Inspecting the above linearized system (35), (36), (37), (38), and (39), we can express $d\log c_{ht}^{\text{comp}}(s)$ as a function of $\{d\log\mu_{ht}(s) - d\log\phi_h\}$:

$$d\log c_{ht}^{\text{comp}}(s) = \mathcal{M}_{(h,:)}\left(d\log\boldsymbol{\mu}_t(s) - d\log\boldsymbol{\phi}\right),$$

where $d\log\boldsymbol{\phi} = \{d\log\phi_h\}_{h=1}^H$, $d\log\boldsymbol{\mu}_t = \{d\log\mu_{ht}\}_{h=1}^H$, and $M_{(h,:)}$ are vectors for each h with coefficients that depend on parameters and shares in the allocations without shocks (see below for the explicit functional form).

We now solve for $d\log\phi_h$. The condition $d\log\tilde{u}_h = d\log\tilde{u}_{\bar{h}}$ can be expressed as

$$\frac{\sum_{s,t}\pi(s)\beta^t c_{ht}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{s,t'}\pi(s)\beta^{t'} c_{ht'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} d\log c_{ht}^{\text{comp}}(s) = \frac{\sum_{s,t}\pi(s)\beta^t c_{\bar{h}t}^{\text{comp}}(s)^{1-\frac{1}{\eta}}}{\sum_{t'}\pi(s)\beta^t c_{\bar{h}t'}^{\text{comp}}(s)^{1-\frac{1}{\eta}}} d\log c_{\bar{h}t}^{\text{comp}}(s)$$

We evaluate this expression at the point with no shocks, $\sigma = 0$, where we know that $c_{ht}^{\text{comp}}(s) = c_h$ for every h, giving us

$$\sum_{t,s}\beta^t\pi(s)d\log c_{ht}^{\text{comp}}(s) = \sum_{t,s}\beta^t\pi(s)d\log c_{\bar{h}t}^{\text{comp}}(s).$$

Substituting

$$\sum_{s,t} \pi(s) \beta^t \sum_{h'} \left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right] d\log \mu_{h't}(s) = \sum_{s,t} \pi(s) \beta^t \sum_{h'} \left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right] d\log \phi_{h'},$$

or,

$$\sum_{h'} \left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right] \frac{\sum_{s,t} \pi(s) \beta^t d\log \mu_{h't}(s)}{\sum_{s,t} \pi(s) \beta^t} = \sum_{h'} \left[\mathcal{M}_{(h,h')} - \mathcal{M}_{(\bar{h},h')}\right] d\log \phi_{h'}.$$

Therefore, a solution is to set

$$d\log \phi_{h'} = \frac{\sum_{s,t} \pi(s) \beta^t d\log \mu_{h't}(s)}{\sum_{s,t} \pi(s) \beta^t}.$$

Substitute this solution into (35) to get

$$\begin{aligned} d\log c_{ht}^{\text{comp}}(s) - d\log c_{\bar{h}t}^{\text{comp}}(s) = & -\eta \left(d\log p_{ht}^{\text{comp}}(s) - d\log p_{\bar{h}t}^{\text{comp}}(s)\right) \\ & - \eta \left(d\log \mu_{ht}(s) - \frac{\sum_{s',t'} \pi(s') \beta^{t'} d\log \mu_{ht'}(s')}{\sum \pi(s') \beta^{t'}} \right. \\ & \left. - d\log \mu_{\bar{h}t}(s) + \frac{\sum_{s',t'} \pi(s') \beta^{t'} d\log \mu_{\bar{h}t'}(s')}{\sum \pi(s') \beta^{t'}}\right). \end{aligned} \tag{40}$$

Combining this with (36), (37), (38), and (39) results in a linear system of equations that pins down $d\log c_{ht}^{\text{comp}}(s)$ in the compensated equilibrium as a function of Ψ, ω, cross-price elasticities in production and consumption (which determine Θ), and the forcing terms

$$d\log \mu_{ht}(s) - \frac{\sum_{t',s'} \pi(s') \beta^{t'} d\log \mu_{ht'}(s')}{\sum \pi(s') \beta^{t'}},$$

all of which can be recovered from the decentralized equilibrium at the status quo. This results in the matrix of interest $\mathcal{M}$, mentioned in the statement of the proposition. We provide an explicit formula for $\mathcal{M}$ in terms of price elasticities and expenditure shares below. But first, we show that the choice of $\bar{h}$ does not affect the results of the proposition.

Showing that the choice of $\bar{h}$ is irrelevant. Here, we also note that the choice of $\bar{h}$ does not affect the approximation formula in (33). First, note that, by inspection, $\sum_{h'} \mathcal{M}_{h,h'} = 0$, since the forcing terms only show up in difference in (40). Hence, adding the same constant to every forcing does not alter $d\log \boldsymbol{c}_t^{\text{comp}}(s)$, which means that $\sum_{h'} \mathcal{M}_{h,h'} = 0$.

We now show that changing $\bar{h}$ has no effect on (33) by using this fact. Start by rearraing:

$$\begin{aligned}\log A &\approx \sum_h \sum_{s,t} q_t(s) p_{ht}(s) c_{ht}(s)\, d\log c_{ht}^{\text{comp}}(s)\, d\log \mu_{ht}(s) \\ &= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s)\, d\log c_{ht}^{\text{comp}}(s)\, d\log \mu_{ht}(s) \\ &= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} \right] d\log \mu_{ht}(s).\end{aligned}$$

We show that this is also equal to

$$\log A^{\text{alt}} = \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} + y_t(s) \right] \left[d\log \mu_{ht}(s) + x_t(s) \right],$$

for any $x_t(s)$ and $y_t(s)$, which is what changing $\bar{h}$ does. To see, consider

$$\begin{aligned}\log A^{\text{alt}} &= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} + y_t(s) \right] \left[d\log \mu_{ht}(s) + x_t(s) \right] \\ &= \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} \right] \left[d\log \mu_{ht}(s) \right] \\ &\quad + \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s)\, y_t(s)\, d\log \mu_{ht}(s) \underbrace{\sum_{h'} \mathcal{M}_{h,h'}}_{=0} \\ &\quad + \sum_{s,t} q_t(s) E_t(s)\, x_t(s) \underbrace{\sum_h \omega_{ht}(s) \sum_{h'} \mathcal{M}_{h,h'} \left[d\log \mu_{h't}(s) - d\log \bar{\mu}_{h'} \right]}_{=0} \\ &\quad + \sum_{s,t} q_t(s) E_t(s) \sum_h \omega_{ht}(s)\, y_t(s)\, x_t(s) \underbrace{\sum_{h'} \mathcal{M}_{h,h'}}_{=0} \\ &= \log A.\end{aligned}$$

The first underbrace follows from the fact that $\sum_{h'} \mathcal{M}_{h,h'} = 0$, the second underbrace follows from the Envelope theorem, which implies that production is statically efficient, so that aggregate real consumption in each date and state does not respond to wedges to a first order, and the last underbrace follows from $\sum_{h'} \mathcal{M}_{h,h'} = 0$.

Explicit Formula for $\mathcal{M}$. To make this more explicit, we now provide the explicit expression for $\mathcal{M}_{hh'}$. For any three positive vectors a, b, and c, define the covariance by

$$Cov_a(b,c) = \frac{\sum_i a_i b_i c_i}{\sum_{i'} a_{i'}} - \frac{\sum_i a_i b_i}{\sum_{i'} a_{i'}} \frac{\sum_i a_i c_i}{\sum_{i'} a_{i'}}.$$

Define the $F \times F$ matrix B with element (f,f) given by

$$B_{f,f'} = (1-\eta)\, Cov_{\omega'}(\Psi_{(:,f)}, \Psi_{(:,f')}) + \sum_{f' \in F} \sum_{j \in H \cup N} \lambda_j (1-\theta_j) Cov_{\Omega_{(j,:)}}(\Psi_{(:,f)}, \Psi_{(:,f')}),$$

where $\Omega_{(j,:)}$ is the jth row of Ω. Define the $F \times H$ matrix D with element (f,h) given by

$$D_{f,h} = Cov_{\omega'}(e_h, \Psi_{(:,f)})$$

where e_h is the h-th basis vector column vector (with hth element equal to 1). Define the $H \times H$ matrix.

$$F = -\eta \Psi_{HF} \left(diag(\lambda_F) - B\right)^{-1} D.$$

where Ψ_{HF} is the $H \times F$ block of Ψ corresponding to households' direct and indirect exposure to each factor and λ_F is the $F \times 1$ vector of static factor shares (the last F elements of λ). Finally, define the $H \times H$ matrix,

$$\mathcal{M} = -\eta \left(F + I\right) - (1-\eta)\mathbf{1}\omega' F + \mathbf{1}\eta\omega'.$$

The element (h,h') of $\mathcal{M}$ is $\mathcal{M}_{hh'}$ in Proposition 7. Note that $\mathcal{M}$ depends on the input-output matrix Ω, expenditure shares ω, and elasticities of substitution in production and consumption, θ_j for $j \in H \cup N$, and the EIS η.

Substituting our expression for $d \log c_t^{\text{comp}}(s)$ into (33) yields the desired expression. The last step is to recognize that at $\sigma = 0$, we have

$$q_t(s) p_{ht}(s) c_{ht}(s) = \frac{\pi(s)(1+r)^{-t} p_h c_h}{\sum_{h',t',s'} (1+r)^{-t'} p_{h'} c_{h'}} = \frac{\pi(s)(1+r)^{-t}}{\sum_{t'} (1+r)^{-t'}} \frac{p_h c_h}{\sum_{h'} p_{h'} c_{h'}}.$$

Hence, we can replace

$$q_t(s) p_{ht}(s) c_{ht}(s) = \frac{\pi(s)(1+r(0))^{-t}}{\sum_{t'} (1+r(0))^{-t'}} \omega_h(0),$$

where $\omega_h(0)$ and $r(0)$ make explicit the dependence of ω_h and r on $\sigma = 0$. We can replace these with the expenditure shares, $\omega_{h0}(\sigma)$, and risk-free rate in the first period, $r(\sigma)$, both in the decentralized equilibrium, since any differences will be third order. □

Derivation of equation (6). Consider first the numerator. The maximization problem

$$\max_{\boldsymbol{c}\in\mathcal{C}} CE^{VOI}(\boldsymbol{c})$$

splits aggregate consumption uniformly across all agents:

$$\boldsymbol{c}_h^* = \frac{\sum_{h'} \boldsymbol{c}_{h'}^0}{H}.$$

The certainty-equivalent of a population-weighted lottery of $\{\boldsymbol{c}_h^*\}$ is

$$CE^{VOI}\left(\{\boldsymbol{c}_h^*\}\right) = \left((1-\beta)\left(1-\tfrac{1}{\eta}\right)\sum_h \frac{1}{H}u(\boldsymbol{c}_h^*)\right)^{\frac{\eta}{\eta-1}} = CE\left(\frac{\sum_{h'} \boldsymbol{c}_{h'}^0}{H}\right),$$

where we used the fact that

$$CE(\boldsymbol{c}_h) = \left((1-\beta)\left(1-\tfrac{1}{\eta}\right)u(\boldsymbol{c}_h)\right)^{\frac{\eta}{\eta-1}}.$$

Consider now the denominator:

$$CE^{VOI}(\{\boldsymbol{c}_h^0\}) = \left((1-\beta)\left(1-\tfrac{1}{\eta}\right)\sum_h \frac{1}{H}u(\boldsymbol{c}_h^0)\right)^{\frac{\eta}{\eta-1}} = \left(\sum_h \frac{CE(\boldsymbol{c}_h^0)^{\frac{\eta-1}{\eta}}}{H}\right)^{\frac{\eta}{\eta-1}}.$$

Taking the ratio of the numerator and denominator yields equation (6).

Derivation of equation (13) Suppose that preferences take the standard functional form in Example 1. In this case, $CE(\boldsymbol{c}_h, \boldsymbol{l}_h)$ is the value of CE that solves

$$\frac{1}{1-\beta}\frac{1}{1-1/\eta}\left(CE^{\gamma}\bar{l}^{1-\gamma}\right)^{1-\frac{1}{\eta}} = u_h(\boldsymbol{c}_h, \boldsymbol{l}_h),$$

so

$$CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \underbrace{\frac{\left[(1-\beta)(1-1/\eta)\right]^{\frac{\eta}{\gamma(\eta-1)}}}{\bar{l}^{\frac{1-\gamma}{\gamma}}}}_{\text{constant}} \times \left(u_h(\boldsymbol{c}_h, \boldsymbol{l}_h)\right)^{\frac{\eta}{(\eta-1)\gamma}},$$

and

$$\sum_h CE(\boldsymbol{c}_h, \boldsymbol{l}_h) = \text{constant} \times \sum_h \left(u(\boldsymbol{c}_h, \boldsymbol{l}_h)\right)^{\frac{\eta}{(\eta-1)\gamma}}.$$

We now calculate $\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*)$ for any allocation $\{\boldsymbol{c}_h^*, \boldsymbol{l}_h^*\}$ in the Pareto-efficient frontier. Under the assumptions of this example,

$$c_{ht}^*(s) = c_h^*, \quad l_{ht}^*(s) = l_h^*, \quad \text{with } \frac{\partial v(c_h^*, l_h^*)}{\partial c_h} = \frac{\partial v(c_h^*, l_h^*)}{\partial l_h},$$

or

$$l_h^* = \frac{1-\gamma}{\gamma} c_h^*.$$

Hence,

$$u_h(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \frac{1}{1-1/\eta} \frac{1}{1-\beta} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\gamma)(\eta-1)}{\eta}} (c_h^*)^{\frac{\eta-1}{\eta}},$$

and

$$\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \frac{[(1-\beta)(1-1/\eta)]^{\frac{\eta}{\gamma(\eta-1)}}}{\bar{l}^{\frac{1-\gamma}{\gamma}}} \times \sum_h \left(u(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*)\right)^{\frac{\eta}{(\eta-1)\gamma}} = \underbrace{\left(\frac{1-\gamma}{\gamma}\frac{1}{\bar{l}}\right)^{\frac{1-\gamma}{\gamma}}}_{\text{constant}} \times \sum_h (c_h^*)^{\frac{1}{\gamma}}.$$

Any allocation $(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*)$ in the Pareto frontier satisfies

$$\sum_h c_h^* = y^* = \sum_h \left(1 - \frac{1-\gamma}{\gamma} c_h^*\right),$$

so

$$\sum_h c_h^* = y^* = \gamma \sum_h 1.$$

Setting $c_h^* = \alpha_h \gamma \sum_h 1$, with $\sum \alpha_h = 1$, we obtain

$$\sum_h CE(\boldsymbol{c}_h^*, \boldsymbol{l}_h^*) = \underbrace{\left(\frac{1-\gamma}{\bar{l}}\right)^{\frac{1-\gamma}{\gamma}} \gamma}_{\text{constant}} \times \sum_h (\alpha_h)^{\frac{1}{\gamma}},$$

which corresponds to equation (13). This measure assigns (weakly) higher values to more unequal weights $\{\alpha_h\}$ since $\gamma \leq 1$.

Appendix B Misallocation Measured via Sum of Compensating Variations

If all household have common homothetic preferences, as in Section 3, then A can be interpreted in terms of the sum of compensating variations from completing financial markets.

To see this, denote the expenditure function for household h by $e_h(\boldsymbol{p}, u_h)$, where $\boldsymbol{p}$ is the state and time contingent vector of prices under complete markets. When markets are complete, the expenditure function is the total wealth the agent needs to reach the indifference curve indexed by u_h. The compensating variation from completing markets for agent h is

$$cv_h = e_h(\boldsymbol{p}, u_h^{\text{complete}}) - e_h(\boldsymbol{p}, u_h^0)$$

where u_h^{complete} is the utility of household h in the equilibrium under complete markets, and u_h^0 is the utility in the initial equilibrium under incomplete markets.

Let A^{KH} denote the ratio of aggregate wealth under complete markets relative to aggregate wealth needed to make households indifferent to the intial equilibrium:

$$A^{KH} = \frac{\sum_h e_h(\boldsymbol{p}, u_h^{\text{complete}})}{\sum_h e_h(\boldsymbol{p}, u_h^0)} = 1 + \frac{\sum_h cv_h}{\sum_h e_h(\boldsymbol{p}, u_h^0)}.$$

If this ratio is bigger than one, then the winners from completing markets can hypothetically compensate the losers with money left-over. The superscript "KH" refers to the fact that this is a Kaldor-Hicks type notion of aggregate efficiency, measured in terms of the money left over after winners compensate the losers.

With homothetic and identical preferences, we can write

$$e_h\big(\boldsymbol{p}, u_h(\boldsymbol{c}_h)\big) = e(\boldsymbol{p})\, CE(\boldsymbol{c}_h),$$

where the certainty-equivalent $CE(\boldsymbol{c}_h)$ is the homogeneous-of-degree-one representation of utility. Hence,

$$A^{KH} = \frac{\sum_h CE\left(\boldsymbol{c}_h^{\text{complete}}\right)}{\sum_h CE\left(\boldsymbol{c}_h^0\right)}.$$

Recall from the proof of Proposition 2 that any Pareto-efficient consumption allocation must satisfy

$$c_{ht}(s) = \alpha_h y_t(s)$$

for some household-specific $\alpha_h \geq 0$ with $\sum_h \alpha_h = 1$. Hence,

$$A^{KH} = \frac{\sum_h CE\left(\boldsymbol{c}_h^{\text{complete}}\right)}{\sum_h CE\left(\boldsymbol{c}_h^0\right)} = \frac{CE(\boldsymbol{y})\sum_h \alpha_h}{\sum_h CE\left(\boldsymbol{c}_h^0\right)} = \frac{CE\left(\sum_h \boldsymbol{c}_h^0\right)}{\sum_h CE\left(\boldsymbol{c}_h^0\right)},$$

where we used $\boldsymbol{y} = \sum_h \boldsymbol{c}_h^0$. By Proposition 2, $A^{KH} = A$.

This result also follows from applying Proposition 1 in Baqaee and Burstein (2025b). This coincidence breaks down if preferences vary across households, as in the international model in Section 5. In this case, A and A^{KH} differ. We use A, rather than A^{KH}, as our measure of misallocation because, as discussed in Baqaee and Burstein (2025b), A^{KH} is unreliable when preferences vary across households.