A Beckerian Theory of Taxation*

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Abstract

We study taxation in matching environments where both equilibrium sorting and the sharing of match surplus are endogenous. Tax reforms lead to complex adjustments in sorting and surplus sharing patterns across the whole economy. We develop a general approach to characterize these equilibrium adjustments using matching elasticities, which are primitive objects that describe how agents' matching choices are affected by pecuniary payoffs in partial equilibrium. Applying this approach to a workhorse model of family formation, we first show that the sorting and surplus sharing responses are closely linked to complementarities in the creation of economic surplus, as first emphasized by Becker (1973). We describe the effects of arbitrary tax reforms on tax revenues, utilities of different agents, and social welfare; design tests to assess Pareto efficiency of existing tax systems; derive formulas that characterize optimal taxes; and propose numerical techniques to compute them. In a calibrated model, we find that optimal marginal taxes are substantially lower and less progressive than the Mirrleesian benchmark.

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1 Introduction

In his seminal work, Gary Becker (Becker (1973)) showed that match formation and within-match resource allocation are determined by the surplus that matches generate and complementarities in producting those gains. Although Becker originally focused on marriage markets, a large subsequent literature has extended his insights to many other settings such as analysis of sorting and matching between firms and workers, workers and cities, corporations and CEOs.

While endogenous sorting, surplus sharing, and complementarities are ubiquitous, effects of taxation in such environments are not well understood. In this paper we develop a general approach to study effects of taxation in a large class of endogenous matching models. We seek to understand what determines costs and benefits of income taxation in such settings, how statutory tax burdens are shared by economic agents, how to assess whether existing taxes are efficient, how should the planner design optimal taxes. We also constrast answers to these questions from those that arise in classical models if taxation in the spirit of Mirrlees (1971).

We first develop our approach in a canonical model of family formation in the spirit Choo and Siow (2006) and Galichon and Salanie (2022). In our baseline model, individuals heterogeneous in labor market productivities form couples based both on pecuniary motives, i.e. utility from consumption and labor that they obtain in those matches, and non-pecuniary preference shocks. As in Becker's work, complementarities in home production affect gains from marriage, sorting of individuals into couples, and sharing of family surplus between spouses. The government cares about redistribution and imposes arbitrary, potentially non-linear taxes on individual earnings to maximize its objective. Equilibrium sorting and the associated pecuniary utilities are determined endogenously, as in standard frictionless matching models with transferable utility.

We show that one of the key objects that determines effects of taxation in this model is the complementarity in home production that captures how the marginal return to effort of one spouse in producing home good is affected by the effort of her spouse. If this return is unaffected, so that there are no complementarities, then the tax analysis is identical to that standard Mirrleesian models of taxation. The effect of any tax reform in that case is summarized by individual labor supply elasticities, and to which we refer to as the direct behavioral effects following Saez (2001). The optimal taxes are characterized by the famous "ABC" formula.

When complementarities are present, i.e. if the marginal return to home effort of one individual is affected in any way by the effort of her spouse, tax reforms have three additional effects. First, partners of individuals facing changes in marginal tax rates may adjust their choices even when their own tax schedules are unchanged; we call this the indirect behavioral effect. Second, tax changes alter incentives to form particular matches, leading agents to reconsider whom to match with and thereby shifting equilibrium sorting patterns; we call this the sorting effect. Third, because sorting shapes the outside options that discipline within-match bargaining, shifts in sorting also change how surplus is divided within matches; we call this the sharing effect.

The indirect behavioral effect can be characterized using techniques from multi-good taxation (e.g., see Chen et al. (2025)). Just as the direct behavioral effect is pinned down by the own labor supply elasticity, the indirect behavioral effect is governed by a cross elasticity of labor supply that measures how a partner's labor supply responds to a change in one's marginal tax rate. With positive complementarities, this cross elasticity is negative: an increase in one spouse's marginal tax rate reduces the partner's labor supply as well, amplifying the distortion. With negative complementarities, the cross elasticity flips sign and the partner's response goes in the opposite direction.

Sorting and sharing effects work fundamentally differently. A tax reform can reshape equilibrium matching patterns and within-family surplus division not only for couples directly affected by the reform, but also for couples whose statutory tax schedules are unchanged; as a result, these effects cannot be studied using off-the-shelf techniques. A central methodological contribution of our paper—one that, as we show in our extension to firm–workers matching, goes far beyond the baseline family-formation setting—is to develop a general approach for describing these two effects.

Our approach is based on what we call *matching elasticities*, which measure how the probability that person A matches with person B changes when some other potential partner C offers A an additional dollar of consumption. Much like behavioral labor supply elasticities—primitives that summarize local properties of preferences and surplus production—matching elasticities are primitives that summarize local properties of the distribution of non-pecuniary shocks. Importantly, matching elasticities, just like behavioral elasticities, are partial-equilibrium objects, which makes them straightforward to compute; for the shock distributions commonly used in applied work, they admit closed-form expressions.

We use matching elasticities to describe how an additional dollar of taxes is shared within a couple, how likely spouses are to dissolve their match, and whom they would match with instead. We show that matching elasticities determine sorting and sharing only through an economy-wide fixed point that links each couple's response to the responses of all other potential matches. We prove that this fixed point can be written as a system of linear equations—akin to the linear systems that arise in production-network models—which delivers a tractable characterization of equilibrium sorting and surplus sharing under arbitrary tax reforms.

Our characterization of sorting and sharing effects reveals several novel insights about effects of taxation under positive and negative complementarities. Under positive complementarities, when the return to home effort by one spouse increases in the effort of her partner, total economic surplus is higher when sorting is more positively assortative. Taxation of high-earners discourages such matches and reduces economic surplus; at the same time it promotes more consumption sharing from high to low earners within families. Transfers to low-earners both encourage more assortative sorting and increase consumption sharing. We describe what determines the relative strengths of these effects and extend analysis to the case of negative complementarities.

Using our characterization, we derive the formula that describe optimal taxes in our environment. In additional to the classical "ABC" terms familiar from the work of Diamond (1998) and Saez (2001), our tax formula has three more terms capturing the indirect behavioral, sorting and sharing effects. We apply this formula to quantify optimal taxes in a calibrated model of family formation. We use findings of Calvo et al. (2024), who estimate complementarities in home production using German time-use survey data of married couples, to discipline the stength of home production complementarities in our model. We find that optimal tax rates are substantially lower and less progressive than in an otherwise identical economy that abstracts from complementarities. This result reflects several forces. Positive complementarities in home production make taxation more distortionary through the indirect behavioral effect. The sorting and sharing effects work in opposite directions across much of the earnings distribution but for high earners losses from sorting effects outweight gains from sharing effects.

In the last part of the paper we show how to apply our techniques to answer a broader set of questions, such as tax incidence or feasibility of Pareto-improving tax reforms, in a wide class of matching model. Our characterization of sorting and sharing effects using matching elasticities extends, with mininal adjustments, to models with non-transferable utilities, broader set of class functions, arbitrary dimensions of heterogeneity, and settings with non-competitive surplus sharing and multi-lateral matching. We also extending our approach to firms and workers matching settings in the spirit of Lamadon et al. (2022), in which heterogeneous firms with monopsony power hire many different heterogenous workers by choosing wages, and individual earnings are subject to income taxation. Despite substantialy differences from our baseline couple formation environment, we demonstrate that the effects of taxation can be characterized using the same toolkit developed in our baseline model.

Our paper is motivated by a large empirical literature that documented assortative matching patterns in a wide variety of settings. For example, more educated and high-earning individuals tend to marry similarly educated and high-earning spouses (Choo and Siow (2006), Fernandez et al. (2005)); more productive employees tend to match with more productive firms (Bonhomme et al. (2019), Lamadon et al. (2022)); and more skilled workers are more likely to reside in cities with a higher share of highly educated residents (Diamond (2016), Diamond and Gaubert (2022)). Our baseline model builds on the canonical framework of marriage markets extensively studied in the family economics literature (Choo and Siow (2006), Chiappori et al. (2017) Galichon and Salanie (2022)), into which we introduce labor income taxation. Several intermediate steps of our analysis-such as the representation of the model with a continuum of types and the characterization of spousal choice probabilities in terms of elasticities-draw on the insights of Dupuy and Galichon (2014).

The closest papers to ours are Sachs et al. (2020), Scheuer and Werning (2017), Ales and Sleet (2016), Ales and Sleet (2022). Like us, Ales and Sleet (2016) and Scheuer and Werning (2017) are motivated by taxation in Beckerian environments; however, both impose assumptions that guarantee perfectly assortative matching (PAM) in equilibrium. Under these assumptions, taxes do not affect sorting and operate only through within-match surplus division. Ales and Sleet (2022) examines optimal taxation in discrete-choice environments with Gumbel-type shocks. These models feature choices shaped by both observed characteristics and idiosyncratic non-pecuniary preferences—an analogy to marriage decisions driven by both pecuniary and non-pecuniary factors. Our framework, however, introduces several additional layers that naturally arise in Beckerian settings: the two- and multi-sided matching, rich non-discrete productivity heterogeneity, intensive margin labor-supply choices, arbitrary distributions of non-pecuniary shocks, and production complementarities. As a result, both the analytical tools and the economic mechanisms differ substantially. Sachs et al. (2020) studies taxation in the presence of general-equilibrium effects and shows that characterizing those effects requires solving for a fixed point of a linear system. In our setting, linear operators that characterize sorting and sharing responses also arise, but the structure of the underlying equations is different.

The normative literature on taxation in Beckerian environments without PAM is relatively scarce. Gayle and Shephard (2019) numerically analyzes optimal taxation in a model of family formation and home production, but in the absence of analytical characterizations is forced to restrict both the matching process and the tax schedule to low-dimensional objects. Dupuy et al. (2020) provides analytical results, but only in an economy with two discrete types.

The paper is organized as follows. In Section 2 we present our baseline environment. In Section 3 we consider effects of taxation in the special case when complementarities are absent. In Section 4 we develop a general approach to characterize how any changes in family surplus affect equilibrium sorting and consumption sharing. In Section 5 these techniques to study effects of taxation. We illustrate quantitative implications of these results in Section 6. In Section 7 we discuss efficiency, general heterogeneity, and taxation of family earnings. In Section 7.4 we illustrate how to extend our techniques to study effects of earnings taxations in the firm-worker matching models in the spirit of Lamadon et al. (2022).

2 The baseline environment

The goal of our paper is to develop a general approach to study taxation in a broad class of economies with endogenous matching and surplus sharing. It will be helpful to develop the key ideas first in a simple model of couple formation and home production. We refer to this model as the baseline environment.

In our baseline environment, heterogeneous individuals form couples, and all heterogeneity is summarized by a unidimensional variable w that captures labor productivity. The economy unfolds in two stages. In the first stage, each individual chooses a partner. When choosing a partner, an individual considers both pecuniary utilities, i.e., utilities from consumption and labor that she can obtain from marrying partners of different types, and idiosyncratic non-pecuniary preference shocks. In the second stage, married couples produce family surplus by working in the market and at home. They pay taxes on their market earnings, purchase consumption goods with after tax income, and share consumption with each other. This determines pecuniary utilities of spouses. In equilibrium, these pecuniary utilities coincide with agents' expectations in the first stage.

The distribution of w is described by a probability measure G. We assume that G satisfies one of two sets of assumptions, to which we refer to as the continuum and finite type cases. In the continuum type case, G has support on some compact interval in $\mathbb{R}_+$ and has density. In the finite type case, G assigns all mass to a finite number of points in $\mathbb{R}_+$. Regardless of whether we consider the continuum or the finite case, we use the same notation $g = (g(w))_w$ to denote the distribution of types: g will be understood to be the probability density in the continuum case and the vector describing measures of agents of each type in the finite case.

2.1 The marriage decision problem

We first describe formally the marriage decision problem of any individual. Let $z = (z(w'))_{w'}$ be the pecuniary utilities that this individual can obtain from marrying spouses of different types and $\epsilon = (\epsilon(w'))_{w'}$ her non-pecuniary prefences for different types. The optimal choice of the partner is described by the solution to

$$\max_{w'} z(w') + \frac{1}{\beta}\epsilon(w'). \tag{1}$$

Here, $\beta \in (0, \infty)$ is a parameter that captures the relative importance of pecuniary motives for partner choice. In the limits, individuals choose partners primarily for non-pecuniary reasons when $\beta \to 0$, and for pecuniary reasons when $\beta \to \infty$.

All individuals of the same type face the same menu of pecuniary utilities z in equilibrium but realizations of non-pecuniary preferences shocks ϵ are idiosyncratic. We use Ψ_w to denote the probability distribution from which shocks ϵ are drawn for type w. We assume that Ψ_w has a positive continuous density and satisfies mild technical conditions stated in the appendix but is general otherwise. In particular, we allow draws across different elements of $(\epsilon(w'))_{w'}$ to be correlated and for the expected values $\mathbb{E}_w\epsilon(w')$ to vary with w'. Let

$$U_w(z) := \mathbb{E}_w[\max_{w'} z(w') + \frac{1}{\beta}\epsilon(w')] \tag{2}$$

denote the ex-ante expected utility of type w who faces a menu of pecuniary utility choices z before non-pecuniary shocks are realized. Function U_w is determined by Ψ_w and is, therefore, a primitive object in our environment.

In applied work, researchers usually choose specifications for the distribution Ψ_w under which U_w has explicit analytical expressions.[1] For example, the canonical logit model, first introduced by McFadden (1974) and extensively used in family economies literature since at least Choo and Siow (2006), assumes that preference shocks are drawn independently for each w' from the EV-I distribution. In this case, U_w takes the form

$$U_w(z) = \frac{1}{\beta}\ln\Big(\sum_{w'}\exp(\beta z(w'))g(w')\Big) + const \tag{3}$$

in the finite type case. The continuum type version of the logit model (see Dupuy and Galichon (2014) or our appendix for the formal description) takes the form

$$U_w(z) = \frac{1}{\beta}\ln\Big(\int \exp(\beta z(w'))g(w')dw'\Big) + const,$$

which is the same expression as (3) but with integrals replacing sums.

We use $\partial_{w'}U_w(z)$ to denote the partial derivative of $U_w(z)$ with respect to $z(w')$. The envelope theorem shows that $\partial_{w'}U_w(z)$ is equal to the ex ante probability that a person of type w would choose partner w'.[2] For commonly used probability distributions Ψ_w, the derivative $\partial_{w'}U_w(\cdot)$ can be written explicitly. For example, in the logit model it is given by

$$\partial_{w'}U_w(z) = \frac{\exp(\beta z(w'))g(w')}{\sum_{\tilde{w}'}\exp(\beta z(\tilde{w}'))g(\tilde{w}')} \quad \text{or} \quad \frac{\exp(\beta z(w'))g(w')}{\int \exp(\beta z(\tilde{w}')))g(\tilde{w}')d\tilde{w}'}, \tag{4}$$

depending on whether one uses the finite or the continuum type specification.

2.2 Pecuniary utilities and complementarities

We now describe how pecuniary utilities are obtained once couples are formed. Suppose all individuals derive utility from market and home goods c, C and disutility from labor supply in the market and at home l, m. Their preferences are given by $c + C - n(l + m)$, where n is an increasing and convex function. Home goods are produced with technology $D(m, m')$, where m and m' are labor supplies at home by the two spouses, and D is increasing, symmetric, and concave. A fraction χ of home good consumption is public within the couple, and $1 - \chi$ is private.

[1] Davis and Schiraldi (2014) and Galichon and Salanie (2022) provide a large list of distributions and implied functional forms of U_w. The analysis of such models is analogous to the logit model that we use for illustrations in the text.

[2] In the case of continuum of types, $(\partial_{w'}U_w(z))_{w'}$ is the Fréchet derivative of $U_w(z)$ in a suitably defined Banach space and describes matching probability densities. We formally define these spaces in the appendix.

Quasi-linearity implies that any efficient allocation of resources within families can be obtained by considering the problem of maximizing the sum of spousal utilities subject to the budget constraint. We can write this problem for family with spouses of types w and w' as

$$\max_{c,c',l,l',m,m'} c + c' + (\chi+1)D(m,m') - n(l+m) - n(l'+m')$$

subject to

$$c + c' \leq wl - T(wl) + w'l' - T(w'l'). \tag{5}$$

The solution to this maximization problem determines optimal labor supplies at home and markets, l, l', m, m', and total consumption of market goods $c+c'$. All points on the Pareto frontier of allocations for the two spouses have the same values of l, l', m, m' and $c+c'$ but differ in how $c+c'$ is divided between the two spouses.

Let

$$N(l,l') := \max_{m,m'} (\chi+1)D(m,m') - n(l+m) - n(l'+m')$$

be the optimal allocation of home labor supply and home consumption conditional on (l,l'). Using function N we can write family maximization problem more succinctly as

$$v(w,w') := \max_{c,c',l,l'} c + c' + N(l,l') \quad \text{subject to (5).} \tag{6}$$

We refer to this v as family surplus. Note that v is a symmetric function, $v(w,w') = v(w',w)$, which captures the fact that labeling of which spouse is 1 and 2 is arbitrary and immaterial for allocations.

In his work, Gary Becker emphasized how complementarities in home production lead to equilibrium sorting and shape the allocation of resources within families. These complementarities are captured in our model by the cross partial derivative $\frac{\partial^2 D}{\partial m \partial m'}$. Our approach to studying taxation does not require us taking a stance on the sign of this derivative but, as we shall see shortly, all departures from the classical theory of taxation in our model are driven by complementarities. We say that there is *no complementarity* in home production if D is separable in the two labor supplies, i.e. $\frac{\partial^2 D}{\partial m \partial m'} = 0$ for all pairs (m,m'); *positive complementarity* if $\frac{\partial^2 D}{\partial m \partial m'} \geq 0$; and *negative complementarities* if $\frac{\partial^2 D}{\partial m \partial m'} \leq 0$. Note that the cross-partial derivative of N takes the same sign as the cross-partial derivative of D. In everything that follows we can take N as the primitive in specification (6) and refer to the cross-partial of N as complementarities.[3]

[3] The analysis is unchanged if complementarities are in consumption of leisure rather than production of home goods. Suppose that each individual has preferences $c + n(\ell, \ell')$ over their own consumption c and leisure ℓ, as well as the leisure of their spouse ℓ', where function n is strictly increasing in ℓ and weakly increasing in ℓ'. Each person has a unit time endowment that can be allocated between work and leisure. Spouses jointly choose their consumption, labor supplies, and leisure to maximize the sum of their utilities subject to the budget constraint (5). This maximization problem can be written as (6) with $N(l,l') := n(1-l, 1-l') + n(1-l', 1-l)$. In this specification, the sign of the cross-partial of N is the same as that of n. Thus, complementarities are positive if spouses enjoy leisurely activities more when they do them together.

2.3 Matching equilibrium

A given tax function T determines family surplus v by (6). We describe equilibrium for any v in terms of two objects: the collection of pecuniary utilities $u = (u_w(w'))_{w,w'}$, where $u_w(w')$ denotes the pecuniary utility that type w obtains in a match with w', and choice probabilities $q = (q_w(w'))_{w,w'}$, where $q_w(w')$ describes the probability with which type w marries w'.

The equilibrium should satisfy three conditions. First, pecuniary utilities u must be feasible, so that the sum of pecuniary utilities in the family is equal to family surplus,

$$u_w(w') + u_{w'}(w) = v(w, w') \;\; \forall w, w'. \tag{7}$$

The pair of pecuniary utilities $(u_w(w'), u_{w'}(w))$ also describes how total consumption $c + c'$, which is determined by the solution to (6), is allocated to spouses. Second, the choice probabilities $(q_w(w'))_{w'}$ must describe optimal choices for each type w given the menu of pecuniary utilities $(u_w(w'))_{w'}$ that they face in equilibrium. Given our discussion in Section 2.1, this optimality condition can be written as

$$q_w(w') = \partial_{w'} U_w(u_w) \;\; \forall w, w'. \tag{8}$$

Finally, the marriage markets must clear. Since non-pecuniary shocks are idiosyncratic, $q_w(w')$ also describes the fraction of w types who choose to marry type w'. As the measure of w types is $g(w)$, the total demand of w types who want to marry w' is $q_w(w')g(w)$. Analogous arguments establish that $q_{w'}(w)g(w')$ is the supply of matches w' available for w. Therefore, the marriage market clearing condition reads

$$q_w(w')g(w) = q_{w'}(w)g(w') \;\; \forall w, w'. \tag{9}$$

These equations allow us to define the marriage market equilibrium as follows

Definition. The equilibrium in the marriage market given family surplus v consists of functions (u, q) such that

(A). Marriage choice probabilities q are optimal given u, i.e., equation (8) holds;

(B). The marriage market clears, i.e., equation (9) holds;

(C). Pecuniary utilities are feasible, i.e., equation (7) holds.

Our notion of equilibrium is mathematically equivalent to the notion of matching stability. The function $f = (f(w, w'))_{w,w'}$ defined by $f(w, w') = q_w(w')g(w)$ describes the joint distribution of matches. f is a probability density function in the continuum case and a stochastic matrix in the case of discrete types, and is symmetric. We use F to denote the cdf of f.[4]

[4]We define equilibrium in terms of pecuniary utilities u and choice probability densities q since understanding the properties of these objects is central to our characterization of optimal taxation. It is common in the literature (e.g., Chiappori and Salanié (2023)) to define matching equilibrium in terms of u and the joint matching probability f. Since f and q are proportional to each other up to the exogenous g, the two ways of defining equilibrium are equivalent.

To build intuition for how matching is determined in equilibrium, it is helpful to consider several special cases. First, consider the case when $\beta \to \infty$. In this case, matching is determined solely by pecuniary motives, as in Becker (1973). Matching is easy to characterize under either positive or negative complementarity. Matching is perfectly positive assortative in the former case and perfectly negative assortative in the latter case; in both cases these equilibrium matching patterns maximize total surplus $\int v dF$. When β is finite, sorting is imperfect. If there are no complementarity and the non-pecuniary shocks are independent of productivities (i.e, the distribution Ψ_w is the same for all w) then matching is random and pecuniary utility $u_w(w')$ is the same for all w'. Either correlation in tastes or complementarities break this result and lead to more complex patterns of sorting and consumption sharing.

3 Taxation without complementarities

In equilibrium, consumption shares u adjust endogenously to clear marriage markets as described by equation (9). Changes in taxes affect family surplus v in (7) and influence both equilibrium sorting and the consumption that individuals obtain in their matches. Understanding how they are influenced by taxes is the main focus of our paper. Before we proceed, it is useful to discuss an important special case of our model – the case of no complementarities.

In the absence of complementarities, the function N is separable and can be written as $N(l, l') = \widetilde{N}(l) + \widetilde{N}(l')$ for some function of a single argument $\widetilde{N}$. Equation (6) then implies that family surplus must be separable as well:

$$v(w, w') = \widetilde{v}(w) + \widetilde{v}(w') \ \ \forall w, w', \tag{10}$$

where $\widetilde{v}$ is defined by

$$\widetilde{v}(w) := \max_{y} y - T(y) - \widetilde{N}\left(\frac{y}{w}\right). \tag{11}$$

Both v and $\widetilde{v}$ depend on the tax schedule T. Let v^{LF} and $\widetilde{v}^{LF}$ be their values in the laissez-faire economy with no taxes, and let (u^{LF}, q^{LF}) be the equilibrium corresponding to v^{LF}. The following lemma shows that in the absence of complementarities, taxes do not affect equilibrium sorting, and all tax burden is absorbed by the consumption of the agent who pays that tax.

Lemma 1. *In the absence of complementarities, the equilibrium* (u, q) *for any* T *satisfies* $q_w(w') = q_w^{LF}(w')$ *and* $u_w(w') = u_w^{LF}(w') + \widetilde{v}(w) - \widetilde{v}^{LF}(w)$ *for all* w *and* w'.

Proof. We need to verify that the constructed (u, q) satisfy Conditions (A)–(C) of the definition of equilibrium. First, observe that $u_w(w') - u_w^{LF}(w')$ is independent of w'. Therefore, the optimal choice of partner w' for person w is unaffected for any realization of the idiosyncratic shock. Thus, Conditions

(A) and (B) hold under taxes T as they did in the laissez-faire equilibrium. To verify (C), observe that for any w and w', we have

$$\begin{aligned} u_w(w') + u_{w'}(w) &= u_w^{LF}(w') + u_{w'}^{LF}(w) + (\widetilde{v}(w) + \widetilde{v}(w')) - (\widetilde{v}^{LF}(w) + \widetilde{v}^{LF}(w')) \\ &= \widetilde{v}(w) + \widetilde{v}(w'), \end{aligned}$$

where the last equality follows from the fact that the laissez-faire equilibrium satisfies (10) and Condition (C). Equation (10) then implies that u satisfies Condition (C). □

This result is remarkable because equilibrium sorting and consumption sharing can be quite complex even without complementarities if the non-pecuniary taste shocks are correlated. Nonetheless, the effect of taxes is very simple: the burden of any tax change is fully absorbed by the statutory taxpayer, and equilibrium sorting is not affected at all.

One implication of this result is that the principles of taxation in our matching model without complementarities are the same as in the classical analysis of income taxation along the lines of Mirrlees (1971). To see this, suppose that the planner wants to choose taxes T to maximize social welfare $W = \int \alpha(w) U_w dG$, where α captures the Pareto weights the planner assigns to different agents. Lemma 1 shows that in the absence of complementarities, we can write W as

$$\begin{aligned} W &= \int \alpha(w) \mathbb{E}_w \left[\max_{w'} \; u_w(w') + \epsilon(w') \right] dG \\ &= \int \alpha(w) \widetilde{v}(w) dG + \int \alpha(w) \mathbb{E}_w \left[\max_{w'} \; u_w^{LF}(w') - \widetilde{v}^{LF}(w) + \epsilon(w') \right] dG. \end{aligned}$$

The value of the last integral in this equation is independent of T. In the absence of complementarities, the optimal earnings choice of any person is independent of the identity of the partner they match with. Thus, the optimal tax problem can be stated as choosing feasible taxes on earnings to maximize $\int \alpha \widetilde{v} dG$, which is isomorphic to the classical problem of optimal income taxation considered by Diamond (1998).

Despite this equivalence result, there are important differences between our and Mirrleesian environments. Our environment allows for home production and economies of scale, complex patterns of sorting and consumption sharing. As our discussion in this section shows, these features per se do not have novel implications for taxation. In contrast, when complementarities are present, as in the economy considered by Becker (1973), taxation has several new effects that are absent in Mirrleesian models. We turn to this case next.

4 Equilibrium responses to surplus changes

In this section, we characterize how equilibrium consumption sharing and sorting respond to changes in family surplus. This characterization plays an important role in our analysis of taxation in subsequent sections. As we show in Section , same techniques also extend to other matching environments, such as matches of spouses with additional dimensions of heterogeneity (e.g., sex differences) or outside options (e.g., singlehood), or matches of workers and firms.

We begin with the finite type case, as the analysis relies on simpler and more familiar mathematical techniques. Section 4.1 defines the key object that allows us to characterize equilibrium responses, to which we refer to as matching elasticities. Section 4.2 shows how matching elasticities summarize equilibrium responses to perturbations of v. Section 4.3 provides an illustration in a two type example. Finally, Section 4.4 extends our approach to the continuum type case.

4.1 Matching elasticities

The choice probability $q_w(w')$ depends both on the pecuniary utility $u_w(w')$ offered in that match and on pecuniary utilities $(u_w(\tilde{w}'))_{\tilde{w}'\neq w'}$ available in alternative matches. Let $\gamma_w(\tilde{w}'|w')$ denote the semi-elasticity of $q_w(w')$ with respect to benefits offered by an alternative match $\tilde{w}'$:

$$\gamma_w(\tilde{w}'|w') = -\partial_{\tilde{w}'} \ln q_w(w') := \frac{-\partial^2_{\tilde{w}'w'} U_w(u_w)}{\partial_{w'} U_w(u_w)} \text{ for all } \tilde{w}' \neq w'. \tag{12}$$

We refer to γ as matching elasticities.

Recall that U_w is an exogenous function determined by the distribution of non-pecuniary shocks Ψ_w. Therefore, γ summarizes local properties of those shocks at the equilibrium allocations. For virtually all distribution used in applied work, γ have explicit analytical expressions in terms of (u, q). For example, in the logit model it is given by $\gamma_w(\tilde{w}'|w') = \beta q_w(\tilde{w}')$.

Matching elasticities determine several auxiliary objects that have transparent economic interpretation and that will help to shed light on the economics of matching and sharing responses to tax reforms. Let $\rho = (\rho_w(w'))_{w,w'}$ and $\delta = (\delta_w(\tilde{w}'|w'))_{w,w',\tilde{w}'}$ be defined by

$$\rho_w(w') := \sum_{\tilde{w}'\neq w'} \gamma_w(\tilde{w}'|w'), \quad \delta_w(\tilde{w}'|w') := \frac{\gamma_w(\tilde{w}'|w')}{\rho_w(w')} \text{ for all } \tilde{w}' \neq w'.$$

The term $\rho_w(w')$ captures how the probability of w matching with w' changes if all types $\tilde{w}' \neq w'$ offer an extra dollar to w. It is easy to see from equation (1) that an extra dollar offered by all potential spouses does not affect the optimal spousal choice. Using this observation it is easy to show that $\rho_w(w')$ also satisfies $\rho_w(w') = \partial_{w'} \ln q_w(w')$ and represents the semi-elasticity of $q_w(w')$ to $u_w(w')$. Furthermore, one can show that γ are positive so that $\delta_w(\cdot|w')$ is the probability distribution that describes whom person w marries if she responds to losing a dollar in surplus in her current match by leaving w'.

Let $\lambda = (\lambda(w, w'))_{w,w'}$ and $\theta = (\theta_w(w'))_{w,w'}$ be defined by

$$\lambda(w, w') := \frac{\rho_{w'}(w)\rho_w(w')}{\rho_w(w') + \rho_{w'}(w)}, \quad \theta_w(w') := \frac{\rho_{w'}(w)}{\rho_w(w') + \rho_{w'}(w)}.$$

We refer to $\lambda(w, w')$ as match sensitivity as it captures the likelihood that a match breaks down if spouses lose a dollar of family surplus. Terms $\theta_w(w')$ and $\theta_{w'}(w)$ capture bargaining weights of two spouses in the couple. We show below that they describe how spouses share changes in family surplus.

It is instructive to consider λ and θ in the logit model. Direct calculations yield $\lambda(w, w') = \beta\frac{(1-q_w(w'))(1-q_{w'}(w))}{(1-q_w(w'))+(1-q_{w'}(w))}$ and $\theta_w(w') = \frac{1-q_w(w')}{(1-q_w(w'))+(1-q_{w'}(w))}$. These equations show that the bargaining power of w is higher if there are more alternative matches, $1 - q_w(w')$. The match sensitivity depends both on the measure of alternative matches and on parameter β that captures the relative importance of pecuniary motives in matching. The more important pecuniary motives are, the higher is the match sensitivity. As the number of types increase, λ and θ converge to β and $\frac{1}{2}$.

4.2 Matching and sharing responses

We consider a perturbation of family surplus in arbitrary direction $\dot{v} = (\dot{v}(w, w'))_{w,w'}$. Let $v + \varepsilon\dot{v}$ be the perturbed surplus and consider the limit as $\varepsilon \to 0$. We use $(\dot{u}, \dot{q})$ to denote equilibrium responses of u and q to this perturbation.

Differentiating Conditions (A)–(C) in direction $\dot{v}$ and rearranging terms yields

$$\dot{u}_w(w') = \theta_w(w')\dot{v}(w, w') + \theta_{w'}(w) \underbrace{\sum_{\tilde{w}' \neq w'} \delta_w(\tilde{w}'|w')\dot{u}_w(\tilde{w}')}_{:=\dot{\pi}_w(w')} - \theta_w(w') \underbrace{\sum_{\tilde{w} \neq w} \delta_{w'}(\tilde{w}|w)\dot{u}_{w'}(\tilde{w})}_{=\dot{\pi}_{w'}(w)}. \tag{13}$$

This equation shows that the response of $\dot{u}_w(w')$ consists of three terms. The first term, $\theta_w(w')\dot{v}(w, w')$, captures the direct effect: the perturbation $\dot{v}$ changes the surplus of couple (w, w') by $\dot{v}(w, w')$, and spouses allocate this extra surplus in proportion to their bargaining weights. The second term, $\theta_{w'}(w)\dot{\pi}_w(w')$, reflects changes in the outside option of w. If w exits the match with w', she can match with any $\tilde{w}' \neq w'$. The vector $(\dot{u}_w(\tilde{w}'))_{\tilde{w}' \neq w'}$ captures changes in these outside options, and $\dot{\pi}_w(w')$ is the change in the expected value that spouse w can obtain conditional on leaving her match with w'. An increase in w's outside option increases her consumption, with the magnitude of the response determined by her partner's bargaining weight $\theta_{w'}(w)$. The third term, $-\theta_w(w')\dot{\pi}_{w'}(w)$, captures symmetric adjustments to changes in the outside options of partner w'.

Equation (13) can also be written as

$$\dot{u}_w(w') = \underbrace{\dot{\pi}_w(w')}_{\text{change in outside option}} + \underbrace{\theta_w(w')}_{\text{bargaining weight}} \underbrace{[\dot{v}(w, w') - \dot{\pi}_w(w') - \dot{\pi}_{w'}(w)]}_{\text{change in net surplus}}. \tag{14}$$

To understand the term in square brackets, note that persons w and w' form a match if their net surplus—the difference between the pecuniary and non-pecuniary benefits of forming that match relative to their outside options—is positive. The term in the square brackets captures how these net gains are affected by perturbation $\dot{v}$. Equation (14) shows that $\dot{u}_w(w')$ equals the change in the value of the outside option of w plus w's share of changes in net surplus, where the share is determined by w's bargaining weight.

In matrix notation, equation (13) can be written as

$$\dot{u} = \Theta\dot{v} + A\dot{u} - B\dot{u}, \tag{15}$$

where matrices Θ, A, and B are defined solely by γ. Using the explicit expressions for A and B from (13), one can verify that both A and B are positive and that $A+B$ is a stochastic matrix. This implies that the spectral radius of $A-B$ is strictly less than one (see the appendix for details), allowing us to write the solution as

$$\dot{u} = \underbrace{(\Theta - A + B)^{-1}}_{:=K}\dot{v} = \Theta\dot{v} + (A-B)\Theta\dot{v} + (A-B)^2\Theta\dot{v} + \cdots. \tag{16}$$

Matrix K in this construction is determined solely by matching elasticities. The geometric series in (16) have a simple economic interpretation. The first term, $\Theta\dot{v}$, captures how agents within couples share the change in family surplus $\dot{v}$ according to their bargaining weights encapsulated in Θ. This sharing changes outside options of spouses. The second term, $(A-B)\Theta\dot{v}$, describes how spouses adjust consumption shares in response to these changes in outside options. These adjustments in turn further change outside options, which requires further adjustments $(A-B)^2\Theta\dot{v}$, and so on.

The analysis of matching responses is very similar. Equilibrium conditions (A) – (C) imply that

$$\frac{\dot{q}_w(w')}{q_w(w')} = \lambda(w,w')[\dot{v}(w,w') - \dot{\pi}_w(w') - \dot{\pi}_{w'}(w)]. \tag{17}$$

Just like sharing responses in equation (14), matching responses depend on changes in net surplus but the responsivenss to these changes is determined by the match sensitivity λ. Since the joint distribution of matches f satisfies $f(w,w') = q_w(w')g(w)$, we can write the left-hand side of (17) also as $\dot{\ln f}(w,w') = \dot{f}(w,w')/f(w,w')$. Following the same steps that we used in showing equation (16), we obtain

$$\dot{\ln q} = \dot{\ln f} = M\dot{v}, \tag{18}$$

where the matrix M is determined solely by γ and has a geometric series representation similar to that of K.

4.3 Illustration in a two-type example

To illustrate these results, suppose that shocks are drawn from the logit distribution and that there are two types, $w \in \{L, H\}$ with $L < H$. Symmetry implies $\theta_L(L) = \theta_H(H) = \frac{1}{2}$, and we must have

$\delta_w(L|H) = \delta_w(H|L) = 1$ for all w. The matrices Θ, A, and B take the form

$$\Theta = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \theta_L(H) & 0 & 0 \\ 0 & 0 & \theta_H(L) & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}, \; A = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \theta_H(L) & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_L(H) \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \theta_L(H) \\ \theta_H(L) & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}.$$

Direct calculations yield that $(A-B)^k = 0$ for all $k > 1$. This shows that with two types, where non-trivial consumption sharing occurs only in HL matches, all equilibrium adjustments occur in a single round.

Solving (16) and (18) explicitly, we obtain $\dot{u}_L(L) = \frac{1}{2}\dot{v}(L,L)$ and $\dot{u}_H(H) = \frac{1}{2}\dot{v}(H,H)$ and

$$\dot{u}_L(H) = \frac{\dot{v}(L,L)}{2} - \theta_L(H)\dot{\varphi}, \quad \dot{u}_H(L) = \frac{\dot{v}(H,H)}{2} - \theta_H(L)\dot{\varphi}, \tag{19}$$

$$\frac{\dot{f}(L,H)}{f(L,H)} = \frac{\dot{f}(H,L)}{f(H,L)} = -\lambda(L,H)\dot{\varphi}, \quad \dot{f}(H,H), \dot{f}(L,L) = -\dot{f}(L,H), \tag{20}$$

where

$$\dot{\varphi} := \frac{1}{2}\left[\dot{v}(L,L) + \dot{v}(H,H) - \dot{v}(L,H) - \dot{v}(H,L)\right]. \tag{21}$$

These equations show that variable $\dot{\varphi}$, that captures changes in the modularity of v, plays the central role in describing equilibrium responses. If v becomes more supermodular, $\dot{\varphi} > 0$, then the surplus in heterogeneous matches HL and LH decreases relative to the surplus in homogeneous matches, HH and LL. Consequently, pecuniary utilities that agents obtain in heterogeneous matches, $u_L(H)$ and $u_H(L)$, fall relative to $u_L(L)$ and $u_H(H)$ as can be seen from equation (19). Similarly, equation (20) shows that the number of heterogeneous matches $f(H,L)$ decreases as higher supermodularity makes heterogeneous matches less attractive. The effects are the opposite if v becomes more submodular, $\dot{\varphi} < 0$.

This insight extends beyond the two type economy or the logit preference shocks. We show in the appendix that responses of $\dot{u}$ and $\dot{\ln f}$ in equation (16) and (18) are captured by a generalized version of $\dot{\varphi}$, extending the intuition of the two-type model to general settings.

4.4 Equilibrium responses with continuum of types

All discussion in this section extends directly to the continuum type case. Equation (12) still defines $\gamma_w(\tilde{w}'|w')$ for $\tilde{w}' \neq w'$. In the continuum case, we extend this definition to $\gamma_w(w'|w')$ as $\gamma_w(w'|w') := \lim_{\tilde{w}' \to w'} \gamma_w(\tilde{w}'|w')$. With this convention, sums in all formulas in the finite type case are replaced with integrals in the continuum case, e.g., $\rho_w(w') := \int \gamma_w(\tilde{w}'|w')d\tilde{w}'$. Equations (14) and (17) remain unchanged, and equation (13) becomes an integral equation of the form[5]

$$\dot{u}_w(w') = \theta_w(w')\dot{v}(w,w') + \theta_{w'}(w)\int \delta_w(\tilde{w}'|w')\dot{u}_w(\tilde{w}')d\tilde{w}' - \theta_w(w')\int \delta_{w'}(\tilde{w}|w)\dot{u}_{w'}(\tilde{w})d\tilde{w}.$$

[5]Such equations are known as Fredholm integral equations of the second kind. The role of these equations in the theory of taxation was first highlighted by Sachs et al. (2020).

The solution to this equation is still characterized by (16), except that Θ, A, and B are integral linear operators rather than matrices. The explicit versions of equations (16) and (18) are

$$\dot{u}_w(w') = \int K_{w,w'}(\tilde{w}, \tilde{w}')\dot{v}(\tilde{w}, \tilde{w}')d(\tilde{w}, \tilde{w}'), \tag{22}$$

$$\frac{\dot{q}_w(w')}{q_w(w')} = \frac{\dot{f}(w, w')}{f(w, w')} = \int M_{w,w'}(\tilde{w}, \tilde{w}')\dot{v}(\tilde{w}, \tilde{w}')d(\tilde{w}, \tilde{w}'),$$

where K and M are depend solely on γ and can be expanded as Neumann series similarly to the geometric series expansion of K in equation (16).

5 Taxation in Beckerian environments

We now apply techniques developed in the previous section to describe effects of taxation. We first consider a two-type economy that we use to highlight novel effects that are not present in Mirrleesian models of income taxation, and illustrate applications of techniques that we developed in the previous section. We then study effects of taxation in our general economy with continuum of types, derive formulas that characterize optimal taxes, and design efficiency tests of existing taxes.

5.1 Taxation in a two-type economy

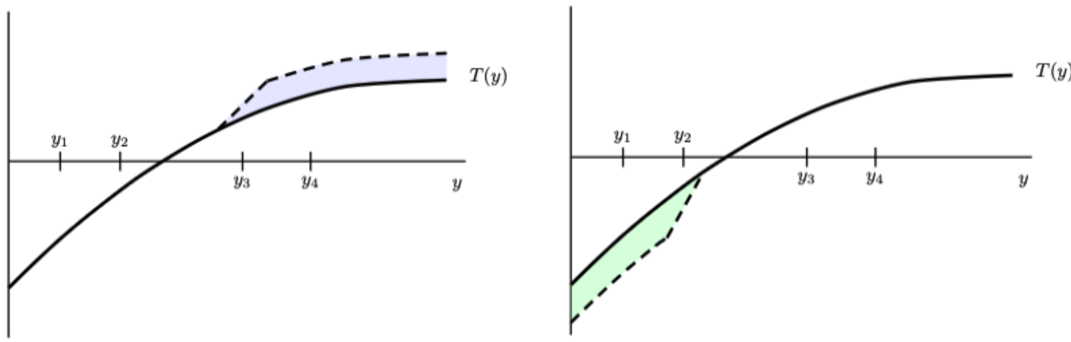
Consider an economy with two types, such as the one we introduced in Section 4.3. Suppose individuals face some tax schedule T. Generically, as long as complementarities are not zero, earnings of any person depend not only on her productivity but also on the productivity of her spouse. Therefore, in the economy with two types there would be four different earnings choices that we can order as

$$y_1 < y_2 < y_3 < y_4. \tag{23}$$

Consider a tax reform that slightly increases the marginal tax rate on earnings in the small neighborhood of income level y_3. This reform increases statutory taxes for incomes $y > y_3$ keeping marginal tax rates for those individuals unchanged. See panel (a) of Figure 1 for a graphical representation of this tax reform.

As shown by Saez (2001), this reform has two effects on tax revenues in Mirrleesian settings. The *mechanical effect* captures the increase in the tax revenues collected from individuals who earn income y_4. The *direct behavioral effect* captures the decrease in tax revenues collected from individuals who earn y_3, because they respond to the increase in the marginal tax rate by reducing their labor supply. The reform reduces utility of the highest earner by the amount of the tax increase, leaving utilities of all other persons unchanged.

In our settings, this tax reform has three more effects. When complementarities are present, the reduction of labor supply of y_3-earners affects the labor supply their spouses even if spouses' taxes



(a) Increasing taxes on the rich, $y > y_3$.

(b) Increasing transfers to the poor, $y < y_2$.

Figure 1: The tax reform in panel (a) increases taxes for individuals with earnings $y > y_3$ by slightly increasing marginal taxes in the neighborhood of y_3. The tax reform in panel (b) increases transfers to individuals with earnings $y < y_2$. The phasing out of this transfers increases effective marginal tax rate for y_2.

are unaffected by the tax reform. This spousal response impacts tax revenues, to which we refer to as the *indirect behavioral effect*. The tax reform also affects incentives whom to marry and how to share family surplus. The individual whose earnings were $y = y_4$ before the reform can avoid the tax increase by leaving her current match and marrying a spouse of a different type than her current partner. This changes sorting patterns, earnings distribution, and tax revenues. We refer to the effect of these changes on tax revenues as the *sorting effect*. Finally, the increase in demand from y_4-earners for new partners also changes how family surplus is shared between different types, to which we refer to as the consumption *sharing effect*. Since these three effects emerge due to complementarities, we refer to them as Beckerian effects.

To build intuition for Beckerian effects we consider the following specification of preferences and taxes. Suppose that the function N takes the form

$$N(l, l') = -\frac{1}{2}(l)^2 - \frac{1}{2}(l')^2 + \phi ll', \tag{24}$$

where parameter ϕ captures complementarity,[6] and that initial taxes are are affine, $T(y) = const + \tau y$ for some $\tau \in (0, 1)$. While simple, this example provides a transparent illustration of main insights that carry over to more general environments.

The optimality condition for labor supply in this economy is $l_w(w') = (1 - \tau)w + \phi l_{w'}(w)$. This equation shows that higher labor supply of the spouse w' encourages labor supply of individual w if complementarities are positive, $\phi > 0$, and discourages if they are negative, $\phi < 0$. Solving these optimality conditions for both partners, we obtain that optimal earnings are

$$y_w(w') = \frac{(1 - \tau)w}{1 - \phi^2}(w + \phi w'). \tag{25}$$

[6]Throughout the discussion, we assume that $\phi \in (-1, 1)$ and small enough to that the first order conditions are interior.

Let $\chi := T(y_H(H)) + T(y_L(L)) - T(y_H(L)) - T(y_L(H))$ be a measure of modularity of tax revenues under the current system.

Suppose that complementarity is positive. In this case equation (25) implies that the ordering of earnings is given by

$$y_L(L) < y_L(H) < y_H(L) < y_H(H), \tag{26}$$

and that both earnings and tax revenues are supermodular in productivities, so that $\chi > 0$. Positive complementarity implies that the increase in the marginal tax rate on earnings y_3 reduces not only earnings $y_H(L)$ but also of her spouse, $y_L(H)$. The increase in the tax level on earnings $y_4 = y_H(H)$ decreases the number of homogeneous matches HH and LL and increases the number of heterogeneous matches HL. This implies that the change in tax revenues due to re-sorting, $-\chi\dot{f}(H,L)$, is negative. Therefore, both the indirect behavioral and sorting effects make it more distortionary to raise taxes from the richest individuals. Consumption sharing effect increases $u_L(H)$ and decreases $u_H(L)$.

We can characterize sorting and sharing effects using the results of Section 4.3. Let normalize the tax increase for y_4 to be one dollar. Then the total tax liability of HH matches increases by two dollars, which changes family surplus in those matches by $\dot{v}(H,H) = -2$. Surpluses in other matches are unchanged, which implies that the change in the modularity of v is $\dot{\varphi} = \frac{1}{2}\dot{v}(H,H) = -1$. Substitute this expression into equations (19) and (20) to find that the sorting effect on revenues is given by $-\chi\lambda(L,H)f(L,H)$ and the sharing effect is $\dot{u}_L(H) = \theta_L(H)$, $\dot{u}_H(L) = -\dot{u}_L(H)$.

Under negative complementarity, the indirect behavioral effect is positive, so that lower labor supply of one individual increases incentives of her spouse to work more. The ordering of earnings is

$$y_L(H) < y_L(L) < y_H(H) < y_H(L) \tag{27}$$

and tax revenue are submodular, $\chi < 0$. Now the tax increase falls on earnings $y_4 = y_H(L)$. This discourages heterogeneous matches and reduces tax revenues due to submodularity of tax revenues. The H types who remain married to L decrease consumption sharing with their spouses, so that $u_L(H)$ falls. All responses can be computed from (19) and (20) by using $\dot{\varphi} = -\dot{v}(H,L) = 1$.[7]

The same analysis applies to other tax reforms. We want to highlight how the sorting effect depends on which part of the tax schedule is being reformed. We saw that the sorting effect is negative for tax increases on the rich, $y > y_3$, both under positive and negative complementarities making such increases more costly and distortionary. Consider an alternative reform that increases transfers to the poor, i.e. for all earnings y that are below the threshold y_2. As additional transfers being phased out, this reform increases effective marginal tax rate for $y = y_2$ (see panel (b) of Figure 1 for a graphical

[7] Recall that v is symmetric and $\dot{v}(H,L) = \dot{v}(L,H)$.

depiction).[8] Note that this reform increases incentives to form homogeneous matches under positive complementarity and to form heterogeneous matches under negative complementarity. In both cases, changes in sorting increases tax revenues, making such reforms less distortionary. The sign of sharing effects remains the same as in the reform in panel (a).

5.2 Effects of taxation in the general economy

We now turn our attention to study effects of taxation in our economy with continuum of types. Let T be a tax function, v be the family surplus corresponding to this T, and (u, q) be the equilibrium for this v. We assume that T is twice differentiable and that optimal earnings choices are unique for all couples. Let

$$R = \int T(y_w(w'))f(w, w')d(w, w') \tag{28}$$

be tax revenues collected in equilibrium. To make our discussion directly comparable to the analysis of Section 3, we assume that the planner evaluates welfare using

$$W = \int \alpha(w)U_w g(w)dw, \tag{29}$$

where Pareto weights α is a positive, decreasing function normalized so that $\int \alpha dG = 1$.

Suppose that the government changes T to $T + \varepsilon\dot{T}$, where $\dot{T}$ is an arbitrary differentiable function and ε is a scalar. We use dots to denote responses of variables to this perturbation as $\varepsilon \to 0$. The envelope theorem applied to (6) shows that the family surplus response is given by

$$\dot{v}(w, w') = -\dot{T}(y_w(w')) - \dot{T}(y_{w'}(w)). \tag{30}$$

We begin by considering the response of tax revenues. Equation (28) implies that $\dot{R}$ is given by

$$\dot{R} = \underbrace{\int \dot{T}(y_w(w'))dF}_{\text{mechanical}} + \underbrace{\int T'(y_w(w'))\dot{y}_w(w')dF}_{\text{behavioral}} + \underbrace{\int T(y_w(w'))\frac{\dot{f}(w, w')}{f(w, w')}dF}_{\text{sorting}}. \tag{31}$$

The first term on the right-hand side is the mechanical effect on tax revenues, the second term captures both direct and indirect behavioral effects, and the third term is the sorting effect.

We want to re-write this equation as an explicit function $\dot{T}$. The mechanical effect is already written in this form. Behavioral effects can be found by considering optimal earnings choices in (6). This choice problem is isomorphic to a multi-good consumer problem with non-linear taxes. In Chen et al. (2025) we develop a general approach to study such problems. Direct application of those results to the particular functional form given in equation (6) yields the following lemma.

[8] It is easy to check that a uniform adjustment in lump-sum taxes does not change modularity of family surplus, $\dot{\phi} = 0$, and such reform has no behavioral, sorting or consumption sharing effects. This implies that the effects of the reform that increases transfers for $y < y_2$ as the same as that of the form that increases taxes for $y > y_2$.

Lemma 2. *The behavioral effect is given by*

$$\text{behavioral} = \underbrace{\int \frac{T'(y_w(w'))}{1-T'(y_w(w'))}\xi^o_w(w')y_w(w')\dot{T}'(y_w(w'))dF}_{direct} + \underbrace{\int \frac{T'(y_{w'}(w))}{1-T'(y_{w'}(w))}\xi^p_w(w')y_w(w')\dot{T}'(y_w(w'))dF}_{indirect}, \tag{32}$$

where $\xi^o_w(w'), \xi^p_w(w')$ *are elasticities of earnings of person* w *married to* w' *with respect to one and partner's after-tax wage rage. These elasticities have explicit expressions in terms of* N *and* T*, and those expressions show that* $\xi^p_{w'}(w)$ *has the same sign as* $\frac{\partial^2}{\partial l \partial l'} N\left(\frac{y_w(w')}{w}, \frac{y_{w'}(w)}{w'}\right)$.

This lemma highlights the general property that the sign of the cross-elasticity is determined by the sign of the cross-partial derivative of N, generalizing the discussion of positive and negative complementarities in Section 5.1. The sign of $\frac{\partial^2}{\partial l \partial l'} N\left(\frac{y_w(w')}{w}, \frac{y_{w'}(w)}{w'}\right)$ captures "local" complementarity at the optimal choices of individuals in the (w, w') couple.

We now turn to the sorting effect. We proceed in two steps. First, using our characterization in equation (22) we re-write the expression for the sorting effect as

$$\text{sorting} = \int \underbrace{\frac{\int T(y_{\tilde{w}}(\tilde{w}'))M_{\tilde{w},\tilde{w}'}(w,w')f(\tilde{w},\tilde{w}')d(\tilde{w},\tilde{w}')}{f(w,w')}}_{:=T^*(w,w')} \dot{v}(w,w')dF.$$

Function $T^*(w, w')$ captures how tax revenues would be affected by changes in equilibrium sorting if all couples in which spouses have productivities w and w' were given an extra dollar of transfers. It is obtained by applying the operator M, which we constructed in Section 4, to the tax function T. An important observation is that T^* does not depend on the direction of the perturbation and is constructed from "partial equilibrium" elasticities. This is helpful because in matching environments different perturbations $\dot{T}$ lead to different "general equilibrium" responses of $\dot{f}$. Without the explicit analytical characterization that we derived in Section 4, one would need to solve a separate fixed point problem for each $\dot{T}$, which would make such brute force approach impractical.

Now combine this expression with equation (30) to obtain explicit characterization of the sorting effect in terms of tax perturbation

$$\text{sorting} = -\int \left[T^*(w,w') + T^*(w',w)\right]\dot{T}(y_w(w'))dF. \tag{33}$$

By substituting (32) and (33) into (31) we find tax revenue response $\dot{R}$ to any tax perturbation $\dot{T}$.

The analysis of welfare response $\dot{W}$ is similar. Using (8) to compute the derivatives of U_w, we obtain

$$\dot{W} = \int \alpha(w)\dot{u}_w(w')\underbrace{q_w(w')g(w)}_{=f(w,w')}d(w,w') = -\int[\alpha^*(w,w') + \alpha^*(w',w)]\dot{T}(y_w(w'))dF, \tag{34}$$

where $\alpha^*(w,w') := \frac{\int \alpha(\tilde{w}) K_{\tilde{w},\tilde{w}'}(w,w') f(\tilde{w},\tilde{w}') d(\tilde{w},\tilde{w}')}{f(w,w')}$ captures the total effect – both from statutory changes in taxes and from equilibrium consumption sharing adjustments – from giving an extra dollar of transfers to couples that consist of spouses of types w, w'.

In Mirrleesian settings, a tax perturbation $\dot{T}$ changes the utility of person w by $-\dot{T}(y_w(w'))$, which has the welfare impact of $-\alpha(w)\dot{T}(y_w(w'))$. We can therefore separate $\dot{W}$ into Mirrleesian and Beckerian terms as follows

$$\dot{W} = \underbrace{-\int \alpha(w)\dot{T}(y_w(w'))dF}_{\text{Mirrleesian}} \underbrace{-\int [\alpha^*(w,w') + \alpha^*(w',w) - \alpha(w)]\dot{T}(y_w(w'))dF}_{\text{sharing}}.$$

5.3 Optimal taxation

We can use results of previous section to characterize optimal taxation. We say that a tax function T is *budget-feasible* if equilibrium tax revenues R are zero. A tax function T is *optimal* if it is budget-feasible and there is no other budget-feasible tax function that yields higher welfare as defined by (29). Throughout, we implicitly assume that the optimal tax function is twice differentiable and that the optimal earning choice of each individual is interior and unique.[9]

Perturbations $\dot{T}$ described in Section 5.2 are not necessarily budget-feasible. However, such perturbation can be made budget-feasible if the government returns all extra revenues $\dot{R}$ as a lump-sum transfer to all individuals. It is straightforward to verify that a uniform lump-sum transfer does not affect consumption sharing, sorting, or the labor supply of any agent. Since the welfare effect of a uniform increase in consumption by all individuals is equal to $\int \alpha dG = 1$, we obtain the following lemma.

Lemma 3. *If tax T is optimal, then*

$$\dot{W} + \dot{R} = 0 \quad \textit{for all } \dot{T}. \tag{35}$$

We can use equation (35) to characterize of the optimal tax schedule. Following Saez (2001), it is more convenient to describe this tax schedule in terms of the distribution of earnings rather than productivities. Let H denote the cdf of earnings under the tax schedule T, and let h denote its density. Define

$$\overline{\xi}^o(y) := \mathbb{E}[\xi^o_w(w')|y_w(w') = y], \quad \bar{\alpha}(y) := \mathbb{E}[\alpha(w)|y_w(w') = y]$$

to be the average labor supply elasticities and Pareto weights of individuals with earnings y. Similarly,

[9] The latter assumption is needed to ensure that the perturbational approach is well-defined. In classical tax models this assumption is closely related to the validity of the first-order approach in the mechanism design formulation of the optimal tax problem. See Chen et al. (2025) for details.

let

$$\bar{T}^*(y) := \mathbb{E}\left[T^*(w,w') + T^*(w',w)|y_w(w') = y\right],$$
$$\overline{\alpha}^*(y) := \mathbb{E}\left[\alpha^*(w,w') + \alpha^*(w',w)|y_w(w') = y\right]$$

be the corresponding averages of terms capturing the sorting and sharing effects. Finally, let

$$\bar{\Upsilon}(y) := \mathbb{E}\left[\frac{T'(y_{w'}(w))}{1-T'(y_{w'}(w))}\frac{\xi^p_w(w')}{\bar{\xi}^o(y)}|y_w(w') = y\right]$$

be the elasticity-weighted average marginal tax rates of the spouses of individuals with earnings y.

We have the following result.

Proposition 1. *The optimal tax schedule satisfies, at every y,*

$$\frac{T'(y)}{1-T'(y)} = \underbrace{\frac{1}{\bar{\xi}^o(y)} \times \frac{1-H(y)}{yh(y)} \times \Big\{\mathbb{E}[1-\bar{\alpha}(\tilde{y})|\tilde{y} > y]}_{Mirrleesian} \tag{36}$$
$$+\underbrace{\mathbb{E}[\bar{\alpha}(\tilde{y}) - \bar{\alpha}^*(\tilde{y})|\tilde{y} > y]}_{sharing}\underbrace{-\mathbb{E}[\bar{T}^*(\tilde{y})|\tilde{y} > y]}_{sorting}\Big\} \quad \underbrace{-\bar{\Upsilon}(y)}_{indirect\ behavioral}.$$

The first line of this equation is classical tax formula, isomorphic to the one obtained by Saez (2001) in his study of optimal nonlinear taxation. The second line highlights the three Beckerian effects that emerge due to complementarities in our model. The sharing and sorting effects for earnings y capture the effects increasing taxes for all earnings $\tilde{y} > y$ similarly to the reform in Figure 1(a). The indirect behavioral effect $\bar{\Upsilon}(y)$ can be written as the sum of two terms: the expected marginal taxes of the spouse of earner multiplied by the ratio of cross- to own-elasticity, and the covariance of the cross-elasticity and the marginal tax rate of the spouse.

Equation (36) not only provides insights about the economic forces that shape optimal taxes but also offers a way to compute optimal taxes numerically. Note that the elasticities ξ, the earnings distribution H, and the surplus sharing and rematching effects $\bar{\alpha}^*$ and $\bar{T}^*$ are endogenous objects that depend implicitly on the equilibrium corresponding to T. Thus, equation (36) can be treated as a fixed-point problem that determines T. In the next section, we describe how it can be solved and discuss its quantitative implications in a calibrated model.

5.4 Efficiency tests

A number of recent papers (e.g., Bierbrauer et al. (2023), Bergstrom and Dodds (2023), Becko and Sztutman (2026)) design tests of (local) efficiency of a given tax schedule T in Mirrleesian settings. Such tests describe whether it is possible to perturb T in a revenue-neutral way to make all agents better off.[10] The general insight of that literature can be summarize in terms of the Laffer curve: the tax schedule is locally efficient if and only if it is impossible to decrease taxes for some agents without reducing total tax revenues of the government.

It is easy to anticipate that Laffer curve tests of efficiency would not be applicable in models of endogenous matching. In such models, tax changes for any one group of agents affects consumption of all agents in the economy through the Beckerian sharing responses. Thus, at the very least, efficiency tests would need to take this sharing response into account. In this section, we show how one can use the operators that we developed in Section 4 to design efficiency tests of a given tax T. Throughout, we assume that the underlying T satisfies the assumptions of Section 5.

We consider three local notions of efficiency

Definition. Taxes are *family efficient* iff $\dot{v} > 0 \implies \dot{R} < 0$. Taxes are *ex-ante individual efficient* iff $\dot{U} > 0 \implies \dot{R} < 0$. Taxes are *ex-post individual efficient* iff $\dot{u} > 0 \implies \dot{R} < 0$.

The three notions of efficiency depends on the unit of observation. Family efficiency means that there is no feasible tax perturbation that increase surplus $v(w, w')$ in all families. While this notion is least satisfactory from welfare point of view, it is helpful to establish the connection with efficiency tests in Mirrleesian settings. Ex-ante individual efficiency refers to the idea that it is impossible to make each individual w better off before they form their matches. Finally, the ex-post individual efficiency requires that there is no feasible perturbation that increase pecuniary utilities $u_w(w')$ in all possible matches.

The impact of the tax perturbation on individual utilities can be written as

$$\dot{u}_w(w') = -\int \underbrace{\frac{K_{w,w'}(\tilde{w}, \tilde{w}') + K_{w',w}(\tilde{w}, \tilde{w}')}{f(\tilde{w}, \tilde{w}')}}_{:=B_{w,w'}(\tilde{w},\tilde{w}')} \dot{T}(y_{\tilde{w}}(\tilde{w}')) f(\tilde{w}, \tilde{w}') \, d(\tilde{w}, \tilde{w}')$$

using the same arguments we employed to derive equation (33). Let $b_w(\tilde{w}, \tilde{w}') := \int B_{w,w'}(\tilde{w}, \tilde{w}') q_w(w') dw'$ and let $\overline{B}_{w,w'}(y)$ and $\bar{b}_w(y)$ denote their expectations conditional on $y_w(w') = y$ defined analogously to other barred variables in Section 5.3. $\overline{B}$ and $\bar{b}$ has clear economic interpretations. $\{\overline{B}_{w,w'}(y)\}_{w'}$ describes how a \$1 tax increase on earnings y affect consumption of person w in matches with every possible spouse w'; $\bar{b}_w(y)$ describes how this tax increase affect expected consumption of person w. Both $\overline{B}$ and $\bar{b}$ constructed directly from the joint distribution f under tax schedule T and matching elasticities γ. Finally, let $F_y(w, w')$ be the distribution of (w, w') conditional on $y_w(w') = y$ and $f_y(w, w')$ be its density.

[10] A closely related "inverse Pareto weight" literature (Blundell et al. (2009), Lockwood and Weinzierl (2016), Hendren (2020)) seeks to find Pareto weights under which the existing tax T is (local) optimum. One can show that T is efficient if and only if the implied inverse Pareto weights are positive. Following most of the literature, we restrict attention to local notion of efficiency in this section. See Werning (2007) for early results about global efficiency, and Chen et al. (2025) for discussion about the connection between local and global notations of efficiency in Mirrleesian settings.

To state our results succinctly, it will be convenient to define function Δ as

$$\Delta(y) := \int_y^\infty (1 - \bar{T}^*(\tilde{y}))dH(\tilde{y}) - \left(\frac{T'(y)}{1-T'(y)} + \bar{\Psi}(y)\right)\bar{\xi}^o(y)yh(y).$$

Function $\Delta(y)$ captures response of tax revenues to a perturbation that raises total taxes for all earnings $\tilde{y} > y$ by a dollar. Function Δ summarizes responses of tax revenues to all possible perturbations $\dot{T}$ since we can write $\dot{R}$ as $\dot{R} = \int \Delta(y)\dot{T}'(y)dy$. Integration by parts imply that $\dot{R} = -\int \Delta'(y)\dot{T}(y)dy$, so that derivative $\Delta'(y)$ captures the effect of the tax reform that decreases taxes for individuals with earnings y. The following proposition provides both efficiency tests and describe a method to obtain inverse Pareto weights.

Proposition 2. *The tax schedule T is family efficient if there exist symmetric Pareto weights $\alpha^f = \left(\alpha^{fam}(w,w')\right)_{w,w'}$ with $\alpha^f \geq 0$, $\int \alpha^f dF = 1$ that satisfy*

$$\int \alpha^f(w,w')f_y(w,w')d(w,w') = -\frac{\Delta'(y)}{h(y)} \quad \forall y. \tag{37}$$

The tax schedule T is ex-ante individual efficient if there exist Pareto weights $\alpha^a = (\alpha^a(w))_w$ with $\alpha^a \geq 0$, $\int \alpha^a dG = 1$ that satisfy

$$\int \alpha^a(w)\bar{b}_w(y)\, g(w)dw = -\frac{\Delta'(y)}{h(y)} \quad \forall y. \tag{38}$$

The tax schedule T is ex-post individual efficient if there exist symmetric Pareto weights $\alpha^p = (\alpha_w(w'))_{w,w'}$ with $\alpha^p \geq 0$, $\int \alpha^p dF = 1$ that satisfy

$$\int \alpha^p_w(w')\bar{B}_{w,w'}(y)f(w,w')d(w,w') = -\frac{\Delta'(y)}{h(y)} \quad \forall y. \tag{39}$$

In this proposition, equations (37), (38), (39) are integral equations that determine implicit social weights α^f, α^a, and α^p as a function of equilibrium objects corresponding to tax T. Once those weights are found, the efficiency tests amount to checking whether corresponding weights are positive.

The test for family efficiency is equivalent to checking whether the tax schedule is located to the left of the peak of the Laffer curve. To see that, observe that $\frac{\Delta'(y)}{h(y)}$ captures the revenue response from cutting taxes for each individual with earnings y by a dollar. If revenues increase from this tax cut then the right hand side of (37) must be negative for some y. Since f_y is positive, weights α^f that solve (37) must be negative for some families (w, w') indicating failure of family efficiency. Conversely, if tax cuts decrease revenues, then the right hand side of (37) must be positive for all y, and α^f that solve (37) must be positive as well.

The simple relationship between efficiency and Laffer curve disappears once we consider welfare of individuals. A tax cut for one spouse can make the other one worse off it is affects consumption sharing within families. Moreover, a tax cut for one family impacts sorting and consumption sharing in families not affected by this reform directly. Therefore, to determine the impact of the tax reform on individual utilities, both ex-ante and ex-post, one needs to find the incidence of the tax reform for all individuals. This is obtained with operators $\bar{b}$ and $\overline{B}$ the ex-ante and the ex-post impact of this incidence.

6 Quantitative illustration

In this section we develop an numerical method that allows one to use formula (36) to find optimal taxes and explore implications of home production complementarities for optimal taxation in a model calibrated to the joint earnings distribution of the U.S. couples.

For our quantitative illustration, we use data from the 2021 March Supplement of the Current Population Survey (CPS), restricting attention to households where both spouses are present, are between the ages of 25 and 65, worked at least 20 weeks in the previous year, and are not self-employed or family workers. To make our calibration transparent, we assume that the reduced-form function N takes the form

$$N(l,l') = -\left(\frac{1}{1+1/\sigma}\,(l)^{\varrho(1+1/\sigma)} + \frac{1}{1+1/\sigma}\,(l')^{\varrho(1+1/\sigma)}\right)^{1/\varrho}, \tag{40}$$

where ϱ is a parameter capturing complementarities,[11] and that individuals in the data face an affine tax function with a marginal rate τ of 30 percent. Following Choo and Siow (2006), we assume that the non-pecuniary shocks are drawn from the logit distribution. These assumptions allow us to recover model parameters from the data using a simple and transparent procedure.

We begin by calibrating preference parameters (σ, ϱ). When preferences are of the form (40), the own and cross elasticities of labor supply are

$$\xi^o = \frac{1}{\varrho(1+1/\sigma)-1} + \frac{(\varrho-1)(\sigma+1)}{\varrho(1+1/\sigma)-1}\mathrm{s}, \;\; \xi^p = \frac{(\varrho-1)(\sigma+1)}{\varrho(1+1/\sigma)-1}\mathrm{s}, \tag{41}$$

where $\mathrm{s} = \frac{(l)^{\varrho(1+1/\sigma)}}{(l)^{\varrho(1+1/\sigma)}+(l')^{\varrho(1+1/\sigma)}}$ is a measure of the share of a given person's labor supply in the family's total labor supply. Equations (41) give a one to one relationship between (σ, ϱ) and the population averages of elasticities $(\mathbb{E}\xi^o, \mathbb{E}\xi^p)$ because $\mathbb{E}s = \frac{1}{2}$. We target $\mathbb{E}\xi^o = \frac{1}{3}$ as value of the average own elasticity of labor supply, which lies in the middle of the range considered by Diamond (1998). For the cross-elasticity, we rely on the results from Calvo et al. (2024), who estimate complementarities in home production using the time use data from the German Socioeconomic Panel. They find that these complementarities are positive across various socio-economic groups. When converted to elasticity form, their estimates imply that the cross elasticity is between one quarter to one half of own elasticity of labor supply (see appendix for details). Given these estimates, we choose a conservative target for

[11] Complementarities are positive when $\varrho > 1$ and negative when $\varrho < 1$. Strict concavity of N requires that $\sigma > 0$ and $\varrho(1+1/\sigma) > 1$.

the average cross-elasticity to be $\mathbb{E}\xi^p = \frac{1}{4}\mathbb{E}\xi^o = \frac{1}{12}$. These values of $(\mathbb{E}\xi^o, \mathbb{E}\xi^p)$ yield

$$\sigma = \mathbb{E}\xi^o + \mathbb{E}\xi^p \approx 0.42, \quad \varrho = \frac{1 + \frac{1}{\mathbb{E}\xi^o - \mathbb{E}\xi^p}}{1 + \frac{1}{\mathbb{E}\xi^o + \mathbb{E}\xi^p}} \approx 1.47.$$

To calibrate the productivity distribution G, we invert the observed earnings distribution of married individuals to recover the underlying distribution of labor market productivity. The relationship between the pair of earnings (y, y') in a family and the pair of productivities (w, w') is given by

$$w = \frac{(y)^{\frac{1/\varepsilon}{1+1/\varepsilon}}}{(1-\tau)^{\frac{1}{1+1/\varepsilon}}} \left[(1+1/\sigma)\mathrm{s}^y\right]^{\frac{\varrho-1}{\varrho(1+1/\sigma)}}, \quad w' = \frac{(y')^{\frac{1/\sigma}{1+1/\sigma}}}{(1-\tau)^{\frac{1}{1+1/\sigma}}} \left[(1+1/\sigma)(1-\mathrm{s}^y)\right]^{\frac{\varrho-1}{\varrho(1+1/\sigma)}}, \tag{42}$$

where $\mathrm{s}^y = \frac{(y)^{\frac{1}{1+1/\sigma}}}{(y)^{\frac{1}{1+1/\sigma}} + (y')^{\frac{1}{1+1/\sigma}}}$ is a measure of the earnings share of a type w who is married to type w'. Equation (41) allows us to construct the pair of productivities (w, w') for each couple and then construct the marginal distribution of productivities. We smooth this distribution by assuming that G is Pareto lognormal (PLN) and choose the three parameters of the PLN distribution to match the mean, the Gini coefficient, and the tail of the empirical marginal distribution of productivities. All three moments have simple analytical expressions for the PLN distribution. We provide additional details and show the goodness-of-fitin the appendix.

Parameter β controls the assortativeness of matches. We set β so that the calibrated model matches the correlation of productivities (w, w') that we recover from the data. The rank correlation of productivities (Kedall's tau) in our sample is 0.18, which yields $\beta = 53$.

6.1 Numerical implementation

We find optimal taxes by iterating on equation (36). We start with a guess of the optimal tax T, which gives the left hand side of (36). Given that T, we find family surplus v, equilibrium (u, q), matching elasticities γ, and operators K and M. We then construct the right hand side of equation (36). If the difference between the two sides of equation (36) exceeds a tolerance bound, we update our guess for T and repeat the procedure until the fixed point of (36) is found.

To implement this approach numerically, we fix an equally log-spaced grid of earnings $\{y_i\}_{i=1}^{Y}$. We specify the candidate tax function in terms of a set of marginal tax rates $\{\tau_i\}_{i=1}^{Y}$ for earnings on this grid. We use cubic splines to extrapolate marginal taxes for all values of earnings and construct (up to a constant that is irrelevant for our analysis) the tax function T. We define the equally log-spaced productivity grid $\{w_i\}_{i=1}^{W}$ where first and terminal points are chosen so that the optimal earnings in couples (w_1, w_1) and (w_W, w_W) are given by y_1 and y_Y. We then compute optimal earnings $\{(y_{w_i}(w_j), y_{w_j}(w_i))\}_{i,j=1}^{W}$ and family surpluses $\{v(w_i, w_j)\}_{i,j=1}^{W}$ for all pairs of productivities.

To find the equilibrium, we start with an observation that (u, q) in the logit model satisfy

$$u_w(w') = \frac{v(w,w') + U_w(u_w) - U_{w'}(u_{w'})}{2}, \quad q_w(w') = \exp\left(\beta \frac{v(w,w') - U_w(u_w) - U_{w'}(u_{w'})}{2}\right) g(w'), \tag{43}$$

where the expected utilities $(U_w(w))_w$ solve

$$U_w(u_w) = \frac{2}{\beta} \ln \left(\sum_{w'} \exp\left(\beta \frac{v(w,w') - U_{w'}(u_{w'})}{2}\right) g(w') \right). \tag{44}$$

We use (44) to find values of expected utilities $\{U_{w_i}(w_i)\}_{i=1}^W$ and then (43) to construct values of (u, q) as well as the matrix of matching elasticities γ on our productivity grid.[12]

We compute the consumption and sharing effects using two different methods. In the first method, we follow the approach we described in Section 4 to construct matrices K and M. This approach is general as it applies to any distribution of non-pecuniary shocks but require inversions of matrices such as $\Theta - A + B$. The matrix $\Theta - A + B$ has dimension $W^2 \times W^2$ and its inversion can be time consuming if the grid of productivities is large.

An alternative method exploits the specific structure of logit shocks and is computationally faster. With logit shocks one can differentiate equation (44) to obtain

$$\dot{U}_w + \sum_{w'} q_w(w') \dot{U}_{w'} = \sum_{w'} q_w(w') \dot{v}(w,w'). \tag{45}$$

We solve this equation for $\dot{U}_w$ and then use (43) to find responses $(\dot{u}, \dot{q})$ to perturbations $\dot{v}$ as follows. Write the left hand side of equation (44) as $(I + Q)\dot{U}$, where I is the identity matrix and Q consists of $\{q_w(w')\}_{w,w'}$ and a find matrix $P = (I + Q)^{-1}$. The solution to (43) can be written as $\dot{U}_w = \sum_{w,w'} p_{\tilde{w}}(w) q_w(w') \dot{v}(w,w')$, where $\{p_w(w')\}_{w,w'}$ are the elements of P. Differentiating (43) and substituting the solution $\dot{U}_w$, we find $\dot{u}$ and $\dot{q}$ as

$$\dot{u}_{\tilde{w}}(\tilde{w}') = \sum_{w,w'} \hat{K}_{\tilde{w},\tilde{w}'}(w,w') \dot{v}(w,w'), \quad \frac{\dot{f}(\tilde{w},\tilde{w}')}{f(\tilde{w},\tilde{w}')} = \sum_{w,w'} \hat{M}_{\tilde{w},\tilde{w}'}(w,w') \dot{v}(w,w'),$$

where matrices $\hat{K}$ and $\hat{M}$ are given by

$$\begin{aligned} \hat{K}_{\tilde{w},\tilde{w}'}(w,w') &= \frac{1}{2} \times \{1(w = \tilde{w}) 1(w' = \tilde{w}') + p_{\tilde{w}}(w) q_w(w') - p_{\tilde{w}'}(w) q_w(w')\}, \\ \hat{M}_{\tilde{w},\tilde{w}'}(w,w') &= \frac{\beta}{2} \times \{1(w = \tilde{w}) 1(w' = \tilde{w}') - p_{\tilde{w}}(w) q_w(w') - p_{\tilde{w}'}(w) q_{w'}(w')\}. \end{aligned}$$

Matrices $\hat{K}$, $\hat{M}$ are not the same as K, M but they satisfy $K\dot{v} = \hat{K}\dot{v}$ and $M\dot{v} = \hat{M}\dot{v}$ for symmetric $\dot{v}$. Since tax perturbations cause only symmetric responses $\dot{v}$, we can use $(\hat{K}, \hat{M})$ instead of (K, M)

[12] In our computations we allow for very large values of w. This makes it preferable to solve equation (44) in two steps. We first introduce a change of variables $\tilde{U}_w := U_w(u_w) - v(w,w)$ and use (44) to solve for $\{\tilde{U}_{w_i}\}_{i=1}^W$ using Newton iterations; we then recover $\{U_{w_i}(w_i)\}_{i=1}^W$ from $\{\tilde{U}_{w_i}\}_{i=1}^W$. Using (44) directly to find $\{U_{w_i}(w_i)\}_{i=1}^W$ causes memory overflow because $\exp(v(w,w))$ increases fast in w.

in all our constructions. The main computational advantage of this alternative method is that it only requires the inversion of the matrix $I + Q$, which has the dimension $W \times W$.[13]

We tested both methods of computing the sorting and sharing effects. Both methods produce the same results but the second one is faster. We use $W = 200$ grid points for productivities, and it takes 577 secs and 32 secs respectively to construct sorting and sharing effects with these two methods on a Macbook laptop using standard Python matrix inversion routines.

Once we solve for behavioral, sorting and sharing responses in the spaces of productivities, we convert them to the space of earnings by building conditional expectations that appear in (36). This step is similar to the approach we developed in Chen et al. (2025) to study optimal taxation in general settings with multi-dimensional heterogeneity, and we leave the details of its implementation for the appendix.

6.2 Quantitative results

We plot optimal taxes corresponding to Pareto weights $\alpha(w) \propto -\exp(-w)$ using solid lines in Figure 2. To isolate the role of complementarities, we compare these results to the optimal taxes in a model that ignores complementarities but that is otherwise calibrated to same moments. In particular, we set $\varrho = 1$ and $\sigma = \frac{1}{3}$ so that the own labor supply elasticity is still $\xi^o = \frac{1}{3}$ (the cross-elasticity is naturally zero), and re-calibrate G to match the same distribution of earnings under the stylized U.S. tax schedule.[14] We plot resulting optimal taxes using dashed lines in Figure 2.

The comparison of solid and dashed lines in Figure 2 reveals that the optimal marginal taxes are lower and less progressive once complementarity is taken into account. The differences are sizable, with the marginal taxes for high earners being about 10 percentage points lower in the model with complementarity. To understand what drives these differences, we plot in Figure 3 the indirect behavioral, sorting, and sharing effects defined in the second line of equation (36).[15]

Figure 3 shows that the indirect behavioral effect is negative, so that it lowers the optimal marginal taxes; the sharing effect is positive and asymptotically decreases to zero for high-earners; and the sorting

[13] We showed in Section 4 that the spectral radius of $A - B$ is less than one (it is $\frac{1}{2}$ in the logit case), which allowed us to write inverse as geometric series $(I - A + B)^{-1} = \sum_{k=0}^{\infty}(A - B)^k$. Interestingly, the spectral radius of Q is equal to one. This implies that the matrix $P = (I + Q)^{-1}$ cannot be written as the standard Neumann series. Instead, P has a representation $P = \sum_{k=0}^{\infty}(-1)^k \left(Q^k - \mathbf{1}g^T\right) + \frac{1}{2}\mathbf{1}g^T$, where $\mathbf{1}g^T$ is due to the fact that Q is a stochastic matrix with the spectral radius of 1. The subtraction of this term removes the unit-eigenvalue component of Q^k ensuring convergence of the infinite sum, and the addition of $\frac{1}{2}\mathbf{1}g^T$ accounts for the action on this invariant subspace.

[14] In the absence of complementarities, the logit model implies that matching is random and cannot match the correlation of productivities in the data. While such correlation can be matched by considering other specifications of Ψ_w, such as the FC-MLN specification used by Galichon and Salanie (2022), we know from Section 3 that the optimal taxes in the absence of complementarities are the same for all Ψ_w. Therefore, we compute optimal taxes by shutting down non-pecuniary shocks entirely, which also makes this approach analogous to the classical approach to study optimal income taxation, such as Diamond (1998) or Saez (2001).

[15] In addition to the three Beckerian effects, the solid and dashed lines in Figure 2 differ because the classical term, $\frac{1}{\xi^o(y)} \times \frac{1-H(y)}{yh(y)} \times \mathbb{E}[1-\bar{\alpha}(\tilde{y})|\tilde{y} > y]$, is endogenous and depends on the underlying tax schedule. Numerically, the difference between values of this term in the two models is small and does not affect our discussion below.

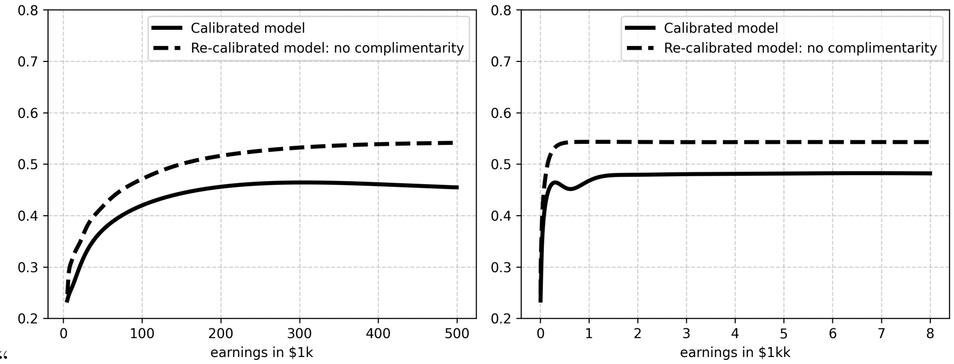


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Figure 2: Optimal tax rates T' . Solid line: calibrated model. Dashed line: re-calibrated model in which we set $\varrho = 1$.

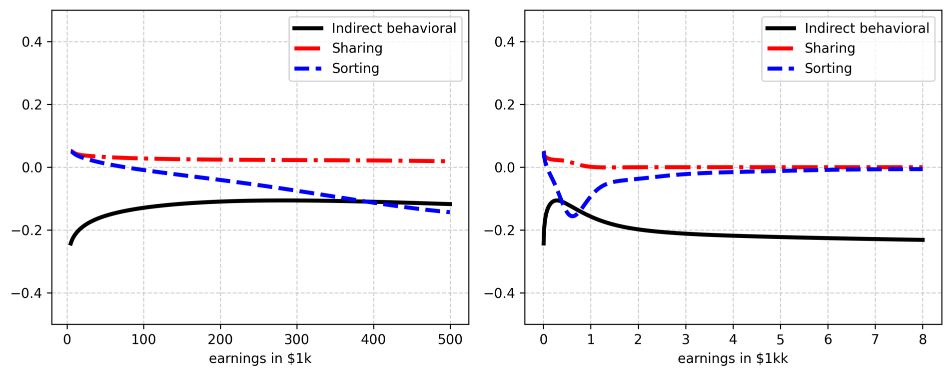

Figure 3: The three Beckerian terms defined in equation (36).

effect is U-shaped, positive for very low earners, negative for all other individuals and asymptotically converges to zero for high-earners. We now explain the economic intution behind these results.

We start with the indirect effect. Under preferences specification (40), the behavioral elasticities are roughly constant so that

$$\bar{\Upsilon}(y) \approx \underbrace{\frac{\mathbb{E}\xi^p}{\mathbb{E}\xi^o}}_{=1/4} \mathbb{E}\left[\frac{T'(y^p)}{1-T'(y^p)}|y^o = y\right],$$

where the expectation denotes expected value of $\frac{T'(y)}{1-T'(y)}$ for the partner of the individual who earns y. Given positive correlation of earnings, $\mathbb{E}\left[\frac{T'(y^p)}{1-T'(y^p)}|y^o = y\right]$ is quantitatively similar to $\frac{T'(y)}{1-T'(y)}$. Consequently, $\bar{\Upsilon}(y) \approx \frac{1}{4}\frac{T'(y)}{1-T'(y)}$, which is roughly the solid line in Figure 3. Thus, the indirect behavioral effect lowers the Mirrleesian value of $\frac{T'(y)}{1-T'(y)}$ by about 25 percent, which makes the marginal taxes both lower and less progressive.

Sorting and sharing effects can be understood from our two-type example in Section 5.1. In that example we showed that the sorting effect increases the cost of taxation of high-earners by making matches less homogeneous under positive complementarities, and decreases the cost of taxes and transfers for low-earners. Consistent with these results, we see in Figure 3 that the sorting effect is negative for high-earners and positive for low-earners. The sharing effect is always positive, in line with our discussion in Section 5.1 of how taxes help to facilitate redistribution of resources from high- to low-productivity individuals in heterogeneous matches. Finally, Figure 3 shows that both the sorting and sharing effects asymptote to zero for very productive agents. This is driven by the fact that the importance of non-pecuniary motives in matching is small for high-productivity agents, which makes sorting and sharing to be asymptotically unresponsive to tax reforms.

7 Application to other Beckerian environments

In our baseline economy agents are identical in all dimensions other than labor market productivity, have quasi-linear, or "perfectly transferable", utility, the government taxes individual earnings, matching is bilateral and surplus sharing is competitive. These features allowed us to streamline the exposition, convey main ideas behind our approach succinctly, and highlight new insights about effects of taxation in a transparent way.

We now describe how to apply the approach that we developed Sections 4 and 5 to a much broader class of endogenous matching economies, and additional insights that emerge in those applications. In Sections 7.1, 7.3, and 7.3 we maintain focus on models of family formation but allow for arbitrary heterogeneity, other forms of taxation, and non-transferable utilities. In Section 7.4 we turn attention to models of firms and workers with multi-lateral endogenous matching and non-competitive surplus sharing.

7.1 Heterogeneity

The general approach that we developed in Sections 4 and 5 did not rely the fact that w was a uni-dimensional variable or that it captured labor productivity. Our approach applies "as is" if w is an arbitrary vector. To see this, let family surplus be

$$v(w, w') := \max_{y,y'} y - T(y) + y - T(y') + N(y, y'; w, w'), \tag{46}$$

where N is some symmetric function in the sense that $N(y, y'; w, w') = N(y', y; w', w)$ and w is a vector of characteristics, which probability distribution is given by G. While this specification is substantially richer than our baseline model, the characterization of sharing and sorting responses $\dot{u}$ and $\dot{f}$ in Section 4 and revenue and welfare responses $\dot{R}$ and $\dot{W}$ in Section 5 is unchanged. Both the optimal tax formulas described in Proposition 1 and efficiency tests described in Proposition 2 also the same as in the baseline model.

The assumption of symmetry for N (which also induces symmetry of v) is merely a statement that the labeling of spouses as 1 and 2 is arbitrary and irrelevant for the analysis. Since distributions G and

Ψ_w are essentially arbitrary, this general specification allows for a wide range of different dimensions of heterogeneity, such as differences in productivities in home work, heterogeneity in elasticities of labor supply, dependence of market or home productivity on sex, education or other observable characteristics, etc. For example, one element of vector w may include a binary sex marker, $s \in \{m, f\}$, and other elements of w (say, corresponding to productivity at market or at home, elasticity of labor supply, etc) may have arbitrary dependence on s. Similarly, either through correlation of pecuniary taste shocks for s' in distribution Ψ_w or in the N function directly, one can embed arbitrarily strong preferences for heterophily along the s dimensions. Thus, this environment allows for essentially any heterogeneity in preferences, productivities, comparative advantage in home production, etc between sexes, and probability distribution of mf, ff and mf matches.[16]

Specification (46) also allows us to highlight the key feature that leads to the departures of tax analysis from that in Mirrleesian models. One can show (see the appendix for the proof) that conclusions of Section 3 hold in our general multi-dimensional environment if $\frac{\partial}{\partial y}N$ is independent of both y' and w'. Thus, Beckerian forces emerge as long as the marginal product of market effort of one spouse depends on some way on either endogenous earnings choices or exogenous characteristics of their partner. On the other hand, Beckerian forces are not directly related to supermodularity of v. For example, if function N takes the form $\tilde{N}(y; w) + \tilde{N}(y'; w') + \Phi(w, w')$ for some functions $\tilde{N}$ and Φ then the cross-partial derivative of v is the same as the cross-partial derivative of Φ but taxes have only Mirrleesian effects.[17]

7.2 Other forms of taxation

In our baseline environment we considered taxation of earnings of individuals. Our analysis extends naturally to other types of taxation, for example, if the government taxes total family earnings as $T(y+y')$. Characterization of sorting and sharing response to perturbation $\dot{v}$ in Section 4 is unaffected. The relationship between surplus changes $\dot{v}$ and tax perturbations $\dot{T}$ is now given by $\dot{v}(w, w') = -\dot{T}(y_w(w')+y_{w'}(w))$ rather than equation (30). Modulo this adjustment, characterization of optimal and efficient

[16] Since we assumed that Ψ_w has density, this formulation would have some positive measure (that can be arbitrarily small) of ff and mm matches. Empirical labor literature often abstracts from possibilities of such matches and models m and f through bilateral matches, where labeling of spouses is not arbitrary but ordered by sex, e.g., f is Spouse 1 and m is Spouse 2 (see, e.g., Choo and Siow (2006), Dupuy and Galichon (2014) or Calvo et al. (2024)). We show in the appendix that our approach extends directly to such environments with minimal and fairly obvious adjustments. For example, in such models there will be sex-specific matching elasticities $\{\gamma_s\}_s$ which one can constructs sex-specific matrices $\{A_s, B_s, \Theta_s\}_s$ that characterize equilibrium responses to perturbations of surplus. The key equation that needs to be solved takes the form $\dot{u}_s = \Theta_s \dot{v} + A_s \dot{u}_s - B_{-s}\dot{u}_{-s}$ for $s \in \{m, f\}$, which is the analogue of (15). One can follow the same steps as in Section 4 to obtain the solution to this equation of the form $\dot{u}_s = K_s \dot{v}$, and find sorting responses of the form $\dot{\ln q_s} = M_s \dot{v}$. We provide additional details in the appendix. See also Section 7.4, in which we discuss taxation in settings in which heterogeneous firms match with heterogeneous workers.

[17] This result can also be seen from our discussion in Section 3. There, we emphasized that even if there are no home production complementarities, $\frac{\partial^2}{\partial l \partial l'}N = 0$, our baseline model can admit arbitrary sorting and surplus sharing via the correlation of taste shocks in Ψ_w. In particular, our Ψ_w imposed not restriction on mean average preference $\mathbb{E}_w \epsilon(w')$. Our baseline specification is equivalent to a model in which family surplus has an additive exogenous component $\Phi(w, w') = \mathbb{E}_w \epsilon(w') + \mathbb{E}_{w'} \epsilon(w)$ and distributions of non-pecuniary shocks $\tilde{\Psi}_w$ are normalized so that $\tilde{\mathbb{E}}_w \epsilon(w') = 0$ for all w, w'.

taxes follows the same steps as Section 5.

Taxation of family earnings introduces additional complementarities in labor supply, which now depend both the cross-partial $\frac{\partial^2 N}{\partial l \partial l'}$ and on the progressivity of the tax schedule T''. Consequently, tax perturbations would generally induce sorting and sharing effects even if $\frac{\partial^2 N}{\partial l \partial l'} = 0$.

It is easy to extend our model to allow an option to remain single, and to introduce a separate tax schedule for earnings of single individuals. The extension of our techniques from Section 4 is immediate to such settings. The model in which both single and married households co-exist combine insights of our baseline model with the results of Golosov and Krasikov (2025), who studied optimal taxation of single and married households in the model of exogenous sorting and surplus sharing. For example, the social planner would generally find it beneficial to encourage marriages because they provide socially valuable redistribution within families.

7.3 Taxation with imperfectly transferable utilities

In our baseline environment, we assumed that utilities were "perfectly transferable", i.e., quasi-linear in consumption. This streamlined discussion because it allowed us to represent all points on the Pareto frontier of efficient allocations within families by a simple maximization problem, equation (6). Our approach to study effects of taxation does not require this assumption and applies to environments with imperfectly transferable utilities.

To make this point as simply as possible, we return to our baseline environment but assume that individual preferences over their consumption c, labor supply l and the labor supply of their partner l' that are given by $V(c + \tilde{N}(l, l'))$, where V is a differentiable, concave function. This model nests our baseline specification with linear V.[18] Let $\tilde{N}_1$ and $\tilde{N}_2$ denote derivatives of $\tilde{N}$ with respect to the first and second arguments.

It is easy to see that within family efficiency requires that labor supplies $(l_w(w'), l_{w'}(w))$ of spouses w and w' must satisfy

$$\begin{aligned} \tilde{N}_1(l_w(w'), l_{w'}(w)) + \tilde{N}_2(l_{w'}(w), l_w(w')) &= -w(1 - T'(wl_w(w'))), \\ \tilde{N}_2(l_w(w'), l_{w'}(w)) + \tilde{N}_1(l_{w'}(w), l_w(w')) &= -w'(1 - T'(w'l_{w'}(w))). \end{aligned} \tag{47}$$

The solution to this system of equations defines $(l_w(w'), l_{w'}(w))$ as an implicit function of T. Let $z_w(w') := wl_w(w') - T(wl_w(w')) + N(l_w(w'), l_w(w'))$ and note that $V(z_w(w'))$ captures utility that person w would obtain if she consumed all her earnings, i.e., if there were no consumption sharing.

There are several mathematically equivalent ways to define equilibrium in this economy. The one that makes the connection to the definition in Section 2 most explicit is the following. Let $v(w, w') :=$

[18] In this case, the function N that appears in (6) is given by $N(l, l') = \tilde{N}(l, l') + \tilde{N}(l', l)$.

$z_w(w') + z_{w'}(w)$. Observe that v is determined by T and pecuniary utilities $(u_w(w'), u_{w'}(w'))$ of spouses satisfy

$$V^{-1}(u_w(w')) + V^{-1}(u_{w'}(w)) = v(w, w'). \tag{48}$$

The equilibrium in marriage markets can be defined just like in Section 2.3, except that in part (C) we replace equation (7) with (48).[19]

The approach that we developed in Sections 4 and 5 applies to this environment with minimal modifications. Sorting and sharing response to perturbation $\dot{v}$ are characterized just like in 4, except that the derivative of condition (C) becomes

$$\frac{1}{V'(u_w(w'))}\dot{u}_w(w') + \frac{1}{V'(u_{w'}(w))}\dot{u}_{w'}(w) = \dot{v}(w, w').$$

This slightly modifies operators K and M that characterize sharing and sorting responses of $\dot{u}, \dot{q}$ but preserves their structure unchanged. The response of $\dot{R}$ and $\dot{W}$ to $\dot{v}$ are then the same as in Section 5.

The relationship between $\dot{v}$ and $\dot{T}$ is no longer given by the envelope theorem, as it was in Section 5, since v is no longer defined by a maximization problem. Instead, we work with the optimality conditions (47) and the definition of v directly. Differentiating these conditions we obtain a response $\dot{v}(w, w')$ as a linear function of $\dot{T}(y_w(w'))$, $\dot{T}(y_{w'}(w))$, $\dot{T}'(y_w(w'))$, $\dot{T}'(y_{w'}(w))$ with explicitly known coefficients. Using this solution, we express $\dot{R}$ and $\dot{W}$ as functions of $\dot{T}$ and $\dot{T}'$. The analysis of optimality and efficiency of taxation is then analogous to those in Sections 5.3 and 5.4.

7.4 Firms, workers, and earnings taxation

Our techniques apply to other environments with endogenous matching and surplus sharing. We illustrate this by considering a model of firm and worker matching in the spirit of Lamadon et al. (2022), or LMS for short. This example also illustrates how to apply our techniques to settings in which matching is not bilateral or surplus sharing is not competitive.

Consider a model where heterogeneous workers match with heterogeneous firms. Let x and a denote heterogeneity of workers and firms. The distribution of workers is given by a probability measure G on $\mathbb{R}_+$ with density g, and the distribution of firms is uniform on $[0, 1]$ interval. Firms set wages for different types of workers. Workers choose which firm to work for based on pecuniary utilities that they derive from wages as well as non-pecuniary preferences. For now, we follow LMS and assume that the labor supply of workers is inelastic.

[19] An alternative way to define equilibrium is by explicitly considering consumption sharing. Let $\omega_w(w')$ denote consumption that spouse w receives from w'. Naturally, the net consumption sharing within a couple is zero so $\omega_w(w')$ and $\omega_{w'}(w)$ must satisfy $\omega_w(w') + \omega_{w'}(w) = 0$. The pecuniary utility $u_w(w')$ is $u_w(w') = V(\omega_w(w') + z_w(w'))$. Therefore, we can define matching equilibrium for given z as a triple (u, q, w) such that (A) equation (8) holds; (B) $u_w(w') = V(\omega_w(w') + z_w(w'))$; and (C) $\omega_w(w') + \omega_{w'}(w) = 0$. This definition of equilibrium is equivalent to the one given in the text.

Let $w_x(a)$ be the wage offered by firm a to worker x. The pecuniary utility of worker x from wage $w_x(a)$ is $\ln(u_x(a))$, where

$$u_x(a) := w_x(a) - T(w_x(a)) \tag{49}$$

denotes worker's after-tax wage and T is the tax schedule. Each worker chooses a firm to work for by solving $\max_{\tilde{a}} \ln u_x(\tilde{a}) + \beta^{-1}\epsilon(\tilde{a})$, where non-pecuniary shocks are drawn from some distribution Ψ_x and β measures the relative importance of pecuniary motives in matching. The ex-ante utility of worker of type x who faces a menu of the after-tax wages $u_x = (u_x(a))_a$ is given by

$$U_x(u_x) := \mathbb{E}_x[\max_{\tilde{a}} \ln u_x(\tilde{a}) + \beta^{-1}\epsilon(\tilde{a})]. \tag{50}$$

The measure of workers of type x who choose firm a is

$$q_x(a) = \partial_a U_x(u_x), \tag{51}$$

where $\partial_a U_x$ is defined by analogy with $\partial_{w'} U_w$ in Section 2.

Distribution Ψ_x is general and can capture a variety on non-pecuniary motives, such as firm heterogeneity in amenities or locations. For example, heterogeneous amenities can be captured by differences in $\mathbb{E}_x\epsilon(a)$ across (x, a). This is equivalent to normalizing all non-pecuniary shocks to have means zero by adding exogenous additive component $\Phi(a, x) = \mathbb{E}_x\epsilon(a)$ to the utility of the worker, which yields amenity specification analogous to the one used by LMS. LMS capture heterogeneity in locations and industry using a nested logit specification of Ψ_x. This specification allows one to write U_x as $U_x(u_x) = \frac{1}{\beta}\ln\left(\sum_{n\in N}\left[\int_{a\in n} e^{\frac{\beta}{\sigma_n}u_x(a)}da\right]^{\sigma_n}\right)$, where N is the (finite) partition of $[0, 1]$ into disjoint nests that capture different markets, i.e., industry*location pairs. Letting $n(a)$ to be the nest of the firm a, we have

$$q_x(a) = \frac{(u_x(a))^{\beta/\sigma_{n(a)}}}{\int_{\tilde{a}\in n(a)} (u_x(\tilde{a}))^{\beta/\sigma_{n(a)}}\, d\tilde{a}} \times \frac{\int_{\tilde{a}\in n(a)} (u_x(\tilde{a}))^{\beta/\sigma_{n(a)}}\, d\tilde{a}}{\sum_{n\in N}\int_{\tilde{a}\in n} (u_x(\tilde{a}))^{\beta/\sigma_n}\, d\tilde{a}},$$

with the two terms capturing substitutability within and across nests.

A worker of type x who works for firm a produces output $\omega(a)x$, where $\omega(a)$ captures productivity of the firm of type a. Firms set wages and hire workers of all types. The profit maximization problem of firm a can be written as

$$\Pi_a = \max_{(w_x(a), q_x(a), u_x(a))_x} \int [\omega(a)x - w_x(a)] q_x(a) g(x) dx \tag{52}$$

subject to (49) and (51). Firm optimality condition is

$$\underbrace{\frac{\omega(a)x - w_x(a)}{w_x(a)}}_{:=m_x(a)} \times \underbrace{\frac{w_x(a)(1 - T'(w_x(a)))}{w_x(a) - T(w_x(a))}}_{:=\tau_x(a)} \times \rho_x(a) = 1, \tag{53}$$

where $\rho_a(x)$ is defined analogously to $\rho_w(w')$ in Section 4.1. In equation (53), $m_x(a)$ captures the optimal wage mark-down, $\tau_x(a)$ is a measure of progressivity of the tax system, and $\rho_x(a)$ is the (semi)-elasticity of supply of labor of worker of type x to firm a.

The equilibrium for a given T can be defined as a collection (u, q, w) that satisfy (49), (51), (53). Utilities of workers and firms U and Π are given by (50) and (52). The density of the joint distribution of matches is $f(a, x) = q_x(a)g(x)$ and the government tax revenues are $R = \int T(w_x(a))f(a, x)d(a, x)$.

Superficially, this model appears to be quite different from the matching environment that we described in Section 2. The firm and worker model features one-to-many matches (each firm hires multiple workers of different types), utility is imperfectly transferable, surplus sharing is determined through wages which are set non-competitively and which are subject to taxation. Despite these differences, the analysis of the effects of taxation in Sections 4 and 5 adapts directly to these settings. We sketch the main steps and draw parallels our discussion in those sections. To streamline our discussion, we focus on the case of nested logit shocks.

Consider the effect of the tax perturbation $T + \varepsilon\dot{T}$. Differentiating equilibrium conditions (49) and (53), one obtains[20]

$$\frac{\dot{u}_x(a)}{u_x(a)} = \tau_x(a)\frac{\dot{w}_x(a)}{w_x(a)} - \frac{\dot{T}(w_x(a))}{u_x(a)}, \quad \frac{\dot{m}_x(a)}{m_x(a)} + \frac{\dot{\tau}_x(a)}{\tau_x(a)} = 0 \tag{54}$$

where $\frac{\dot{m}_x(a)}{m_x(a)}$ and $\frac{\dot{\tau}_x(a)}{\tau_x(a)}$ are given explicitly in terms of $\dot{T}$ by

$$\frac{\dot{m}_x(a)}{m_x(a)} = -\left(1 + \frac{1}{m_x(a)}\right)\frac{\dot{w}_x(a)}{w_x(a)},$$

$$\frac{\dot{\tau}_x(a)}{\tau_x(a)} = \left(1 - \frac{w_x(a)T''(w_x(a))}{1 - T'(w_x(a))} - \tau_x(a)\right)\frac{\dot{w}_x(a)}{w_x(a)} + \frac{\dot{T}(w_x(a))}{u_x(a)} - \frac{\dot{T}'(w_x(a))}{1 - T'(w_x(a))}.$$

Using equation (54) one can write responses $\frac{\dot{u}_x(a)}{u_x(a)}, \frac{\dot{w}_x(a)}{w_x(a)}$ as a linear function of $\dot{T}(w_x(a))$ and $\dot{T}'(w_x(a))$. From this solution, we construct sorting response $\dot{\ln q}$ as

$$\frac{\dot{q}_x(a)}{q_x(a)} = \rho_x(a)\frac{\dot{u}_x(a)}{u_x(a)} - \int \gamma_x(\tilde{a}|a)\frac{\dot{u}_x(\tilde{a})}{u_x(\tilde{a})}d\tilde{a},$$

where $(\gamma_x(\tilde{a}|a))_{x,a,\tilde{a}}$ are matching elasticities defined by analogy with $(\gamma_w(\tilde{w}'|w'))_{w,w',\tilde{w}'}$ in Section 4. Solving this integral equation one can find both $\frac{\dot{q}_x(a)}{q_x(a)}$ and $\frac{\dot{f}(a,x)}{f(a,x)}$ as

$$\dot{\ln q} = \dot{\ln f} = M\dot{T} + L\dot{T}',$$

which is the analogue of (18) and M and L are linear operators known explicitly in terms of matching elasticities and other equilibrium objects corresponding to the tax schedule T.

[20] In deriving these expressions we used the fact that under nested logit structure matching semi-elasticity $\rho_w(a)$ does not depend on utility in $\tilde{a} \neq a$ matches, which leads to $\dot{\rho}_x(a) = 0$. For arbitrary distribution of non-pecuniary shocks, these expressions would also have a term $\dot{\rho}_x(a)$ that is defined in terms of super-elasticities, i.e., the semi-elasticities of γ. Those super elasticities can be constructed by direct analogy with the way we constructed γ in Section 4.

The analysis of the effects of taxation on utilities $\dot{U}_x$, profits $\dot{\Pi}_a$, and tax revenues $\dot{R}$ parallels that of Section 5. For example, the response of tax revenues to this perturbation is given by

$$\dot{R} = \int \dot{T}(w_x(a))f(a,x)d(a,x) + \int T(w_x(a))\frac{\dot{f}(a,x)}{f(a,x)}f(a,x)d(a,x)$$
$$+ \int T'(w_x(a))\dot{w}_x(a)f(a,x)d(a,x).$$

The first two terms in this equation correspond to the mechanical and sorting effects in equation (31). The last term is the sharing effect. This term did not appear in (31) in family taxation settings because consumption sharing was not taxes directly in that environment. There are no behavioral terms in this equation because labor supply is exogenous.

It is easy to add Mirrleesian elements to this analysis by endogenizing the labor supply. Suppose preferences are given by $\ln(c - N(l))$, where c is consumption and l is labor supply, and the budget constraint is $c = w_x(a)l - T(w_x(a)l)$. Let

$$u_x(a) = \max_l w_x(a)l - T(w_x(a)l) - N(l) \tag{55}$$

be consumption of individual adjusted for the disutility of labor. This maximization problem determines the optimal labor supply of worker x in firm a via the optimality condition

$$w_x(a)\left[1 - T'\left(w_x(a)l_x(a)\right)\right] = N'\left(l_x(a)\right). \tag{56}$$

The firm wage setting optimality condition can be written as

$$1 = m_x(a)l_x(a)\tau_x(a)\rho_x(a) + m_x(a)\xi_x(a), \tag{57}$$

where $\xi_x(a) = \frac{\partial \ln l_x(a)}{\partial \ln w_x(a)}$ is the elasticity of labor supply of that is determined by (56). The equilibrium for a given T can be defined as a collection (u, q, w, l) that satisfy (51), (55), (56), (57). The analysis proceeds as in the case with inelastic labor supply, except that $\dot{R}$ now includes behavioral labor supply responses familiar from the Mirrleesian literature.

8 Conclusion

In this paper, we developed new techniques to study the effects of taxation in environments with complementarities and endogenous matching. A key feature of such settings is that tax changes affecting one group of agents can propagate through equilibrium adjustments in sorting and income, thereby influencing outcomes for all agents in the economy. We developed linear operator techniques that allow us to characterize this dependence, and study efficiency and optimality of tax functions.

Due to space constraints, we restricted our analysis to models with frictionless matching. However, we conjecture that the techniques developed in this paper can be extended to broader classes of endogenous matching environments such as matching with frictions along the lines of Shimer and Smith (2000). We leave these extensions to future work.

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